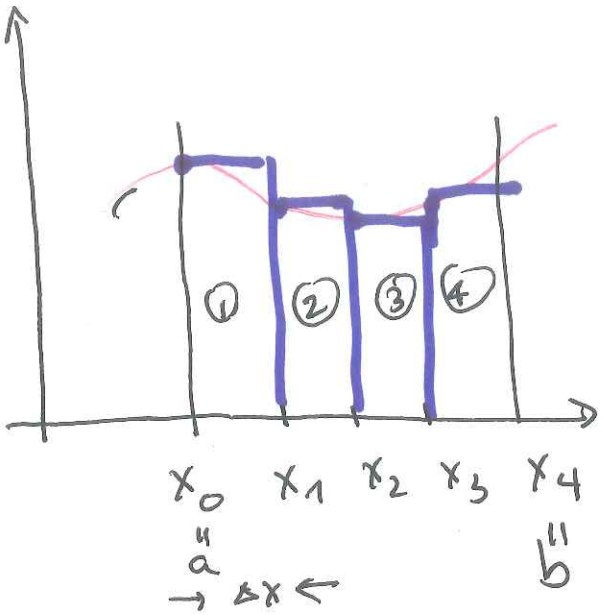
$$\int_a^b f(x)\,dx$$

Left



$N = 4$

$\Delta x = \frac{b-a}{4}$

$x_i = a + i\,\Delta x$

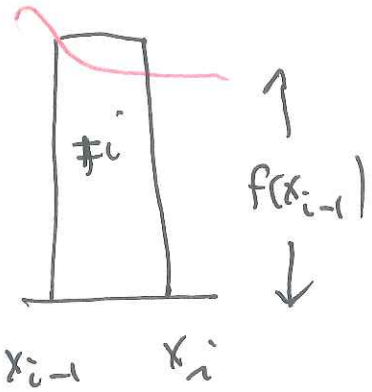
① $\Delta x\, f(x_0)$

② $\Delta x\, f(x_1)$

③ $\Delta x\, f(x_2)$

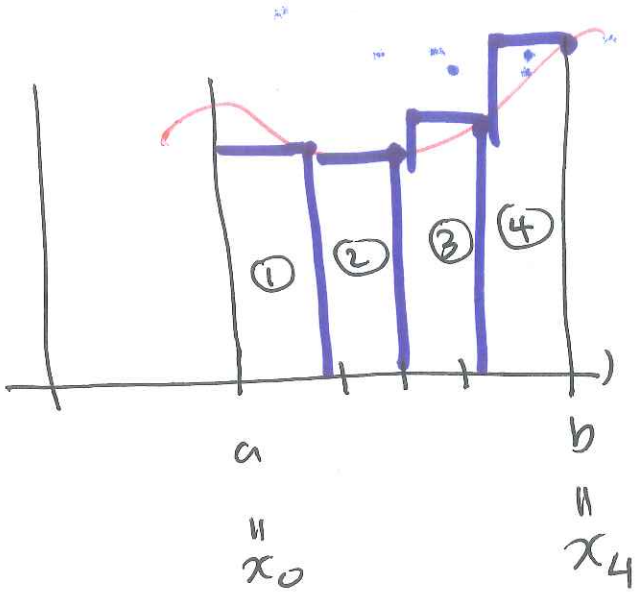
④ $\Delta x\, f(x_3)$

⑤ $\Delta x\, f(x_4)$



$$\int_a^b f(x)\,dx$$

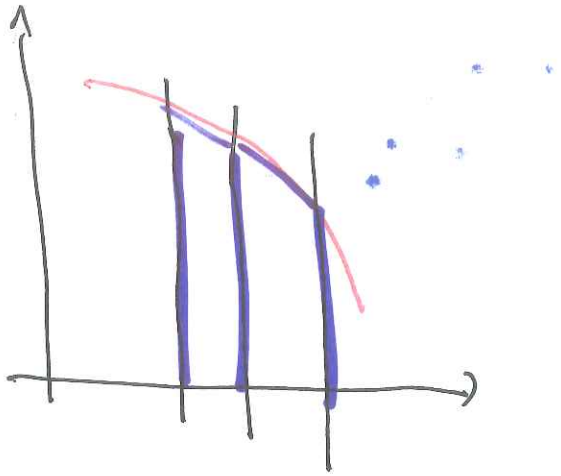
Right



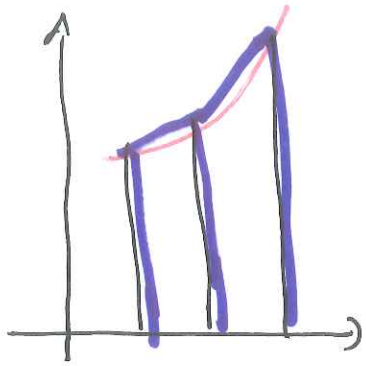
$N = 4$

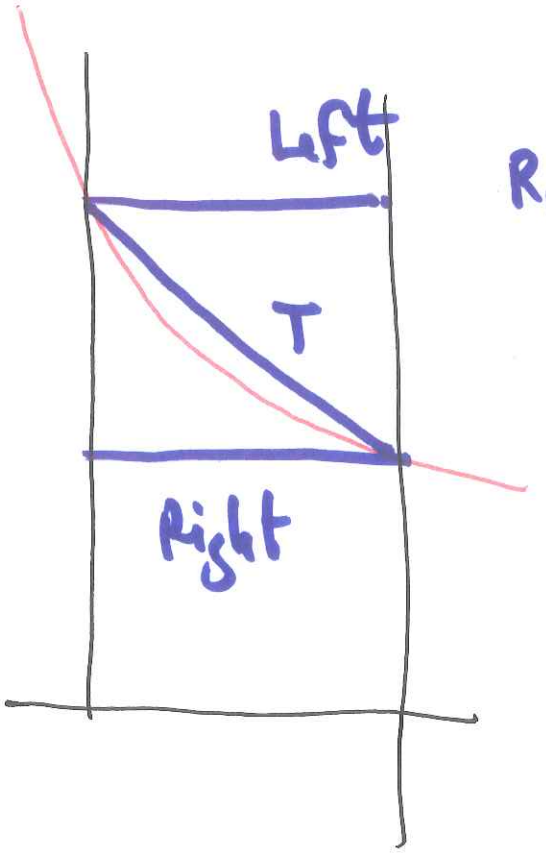
$$R_4 = \Delta x\big(\underset{①}{f(x_1)} + \underset{②}{f(x_2)} + \underset{③}{f(x_3)} + \underset{④}{f(x_4)}\big)$$

f concave down



f concave up





$R_n \leq I \leq T \leq L_n$

$$\int_{-h}^{h} (ax^2 + bx + c)\, dx =$$

$$\left[a\frac{x^3}{3} + b\frac{x^2}{2} + cx \right]\Big|_{-h}^{h} =$$

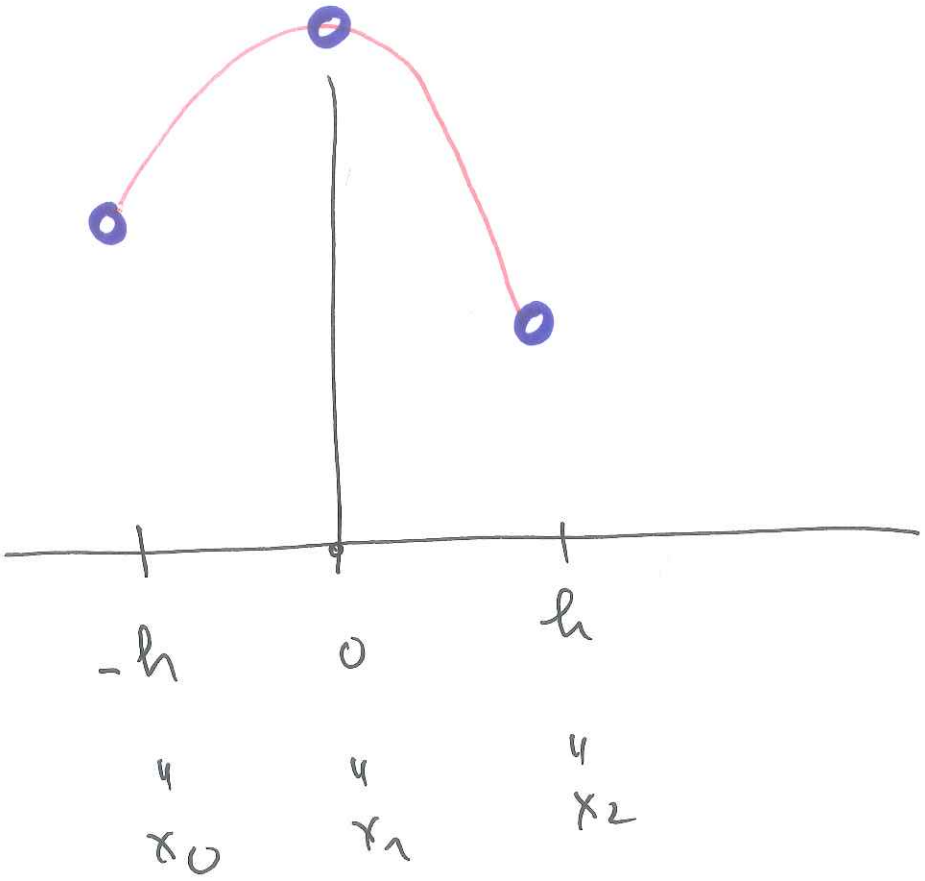
$$\left(a\frac{h^3}{3} + b\frac{h^2}{2} + ch\right)$$

$$+$$

$$-\left(a\frac{(-h)^3}{3} + \frac{bh^2}{2} - ch\right) =$$

$$+$$

$$= 2a\frac{h^3}{3} + 2ch$$



$$f(x) = ax^2 + bx + c$$

$$f(x_0) = f(-h) = ah^2 - bh + c$$

$$f(x_1) = f(0) = c$$

$$f(x_2) = f(h) = ah^2 + bh + c$$