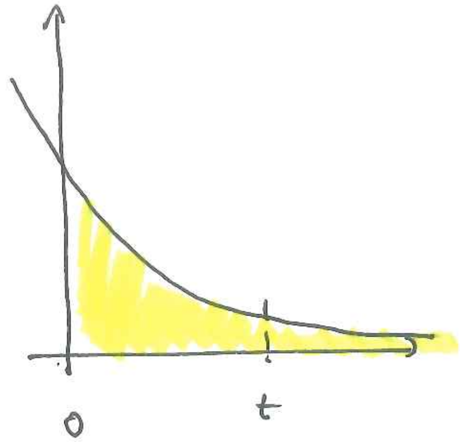
Type I

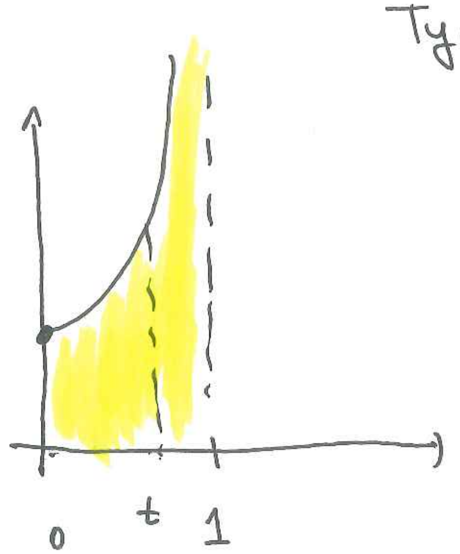


$$f(x) = e^{-x}$$

$$\int_0^\infty e^{-x}\,dx =$$

$$\lim_{t\to\infty} \int_0^t e^{-x}\,dx$$

Type II



$$f(x) = \frac{1}{\sqrt{1-x}}$$

$$\int_0^1 \frac{1}{\sqrt{1-x}}\,dx =$$

$$\lim_{t\to 1^-} \int_0^t \frac{1}{\sqrt{1-x}}\,dx$$

$$\lim_{t\to\infty}\left(3-\frac{2}{t}\right) = 3-0=3$$

$$\lim_{t\to\infty}(1-e^{-t}) = 1-0=1$$

$$\lim_{x\to 1^-}\sqrt{1-x} = 0$$

Type I $\qquad \displaystyle\int_1^\infty \frac{1}{x^2}\,dx$

$$A(t) = \int_1^t \frac{1}{x^2}\,dx = \left[-\frac{1}{x}\right]\Big|_1^t$$

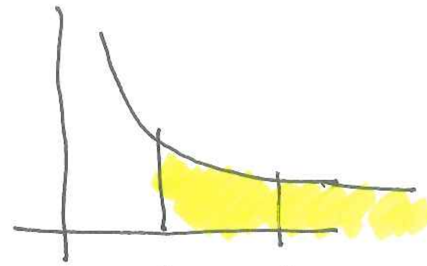
$$= -\frac{1}{t} - (-1)$$

$$A(t) = 1-\frac{1}{t} = \frac{t-1}{t}$$

$$\lim_{t\to\infty} A(t) = 1$$

*

$$\int_1^\infty \frac{1}{x}\,dx$$



$$A(t) = \int_1^t \frac{1}{x}\,dx$$

$$= [\ln x]\Big|_1^t$$

$$= \ln t - \ln(1)$$

$$= \ln t$$

$$\lim_{t\to\infty} A(t) = +\infty$$

$$\int_1^\infty \frac{1}{x}\,dx \quad \underline{\underline{\text{diverges}}}$$

$$A(t) = \int_0^t e^{-x}\,dx = [-e^{-x}]\Big|_0^t$$

$$= (-e^{-t}) - (1)$$

$$= 1 - e^{-t}$$

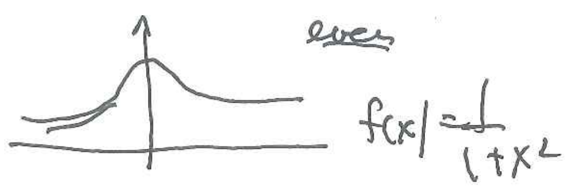
$$A(t) = \int_0^t e^{-x}\,dx$$

$$= 1 - e^{-t}$$

$$\lim_{t \to +\infty} A(t) = \lim_{t \to \infty} (1 - e^{-t})$$

$$= 1$$

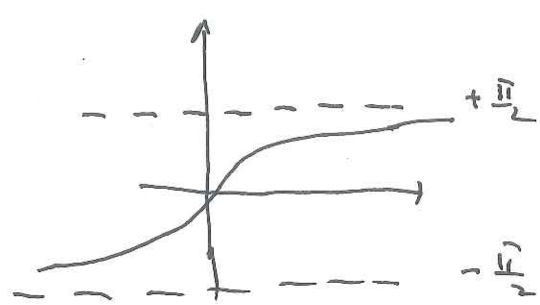
$$\boxed{\int_0^{\infty} e^{-x}\,dx = 1}$$



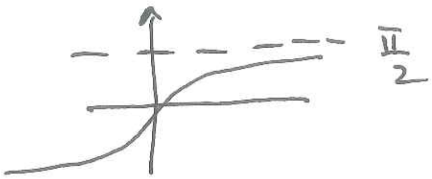
$$\int_{-\infty}^{\infty} \frac{1}{1+x^2}\,dx =$$

$$2\int_0^{\infty} \frac{1}{1+x^2}\,dx$$

$$A(t) = \int_0^t \frac{1}{1+x^2}\,dx$$



arctan(x)

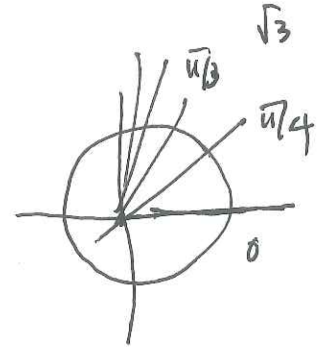


$$A(t) = \int_0^t \frac{1}{1+x^2}\,dx = \arctan(t) - \arctan(0)$$

$$= \arctan(t) - 0$$

$$\lim_{t\to\infty} A(t) = \lim_{t\to\infty} \arctan(t)$$

$$= \frac{\pi}{2}$$



$$\int_0^\infty \frac{1}{1+x^2}\,dx = \frac{\pi}{2}$$

$$\int_{-\infty}^\infty \frac{1}{1+x^2}\,dx = \pi$$

$$\int_a^b f(x)\,dx = -\int_b^a f(x)\,dx$$

$$\int_0^1 \frac{1}{\sqrt{1-x}}\,dx$$

$$A(t) = \int_0^t \frac{1}{\sqrt{1-x}}\,dx$$

$$u = 1-x$$
$$du = -dx$$

when $x=$	then $u = 1-x$
0	1.
t	$1-t$.

$$A(t) = \int_1^{1-t} \frac{1}{\sqrt{u}}\,(-du)$$

$$= \int_{1-t}^1 \frac{1}{\sqrt{u}}\,du$$

$$= \left[2\sqrt{u}\right]\Big|_{1-t}^{1}$$

$$\boxed{A(t) = 2 - 2\sqrt{1-t}}$$

$$\lim_{t\to 1^-} A(t) = 2$$

$$\int_0^1 \log x \, dx = \lim_{t \to 0^+} \int_t^1 \log x \, dx$$

$$A(t) = \int_t^1 \log x \, dx$$

Remember $\int \log x \, dx = x \ln x - x + C$

$$A(t) = [x \ln x - x]\Big|_t^1$$

$$= (1 \ln 1 - 1) - (t \ln t - t)$$

$$= (-1) - (t \ln t - t)$$

$$\lim_{t \to 0^+} \left[(-1) - t \ln t + t \right]$$

$$\downarrow \quad \downarrow \quad \downarrow$$

$$-1 \quad 0 \quad 0$$

$$\lim_{t \to 0} t \ln t = \lim_{t \to 0} \frac{\ln t}{1/t} = \lim_{t \to 0} \frac{1/t}{-1/t^2}$$

$$= \lim (-t) = 0$$