$p = 1$ $\quad \displaystyle\int_1^\infty \frac{1}{x}\,dx = \lim_{t\to\infty} \int_1^t \frac{1}{x}\,dx$

$$= \lim_{t\to\infty} \left(\ln t - \cancel{\ln 1}^{\,0}\right)$$

$$= +\infty$$

$p \neq 1$ $\quad \displaystyle\int_1^\infty \frac{1}{x^p}\,dx = \lim_{t\to\infty} \left(\int_1^t \frac{1}{x^p}\,dx \right)$

Recall $\displaystyle\int \frac{1}{x^p}\,dx = \int x^{-p}\,dx = \frac{x^{1-p}}{1-p} + C$

$$\int_1^\infty \frac{1}{x^p}\,dx = \lim_{t\to\infty} \left(\frac{t^{1-p}}{1-p} - \frac{1}{1-p} \right)$$

$p > 1$

$p = 2$ $\quad \displaystyle\int_1^\infty \frac{1}{x^2}\,dx = \lim_{t\to\infty} \left(\frac{t^{-1}}{-1} - \frac{1}{-1} \right)$

$$= \lim_{t\to\infty} \left(1 - \frac{1}{t} \right) = 1$$

$$\int_1^\infty \frac{1}{x^p}\,dx \quad \text{converges?}$$

$$\int_1^t \frac{1}{x^p}\,dx = \frac{1}{p-1}\left(1 - \underline{\underline{t^{1-p}}}\right)$$

$p > 1$ CONVERGES

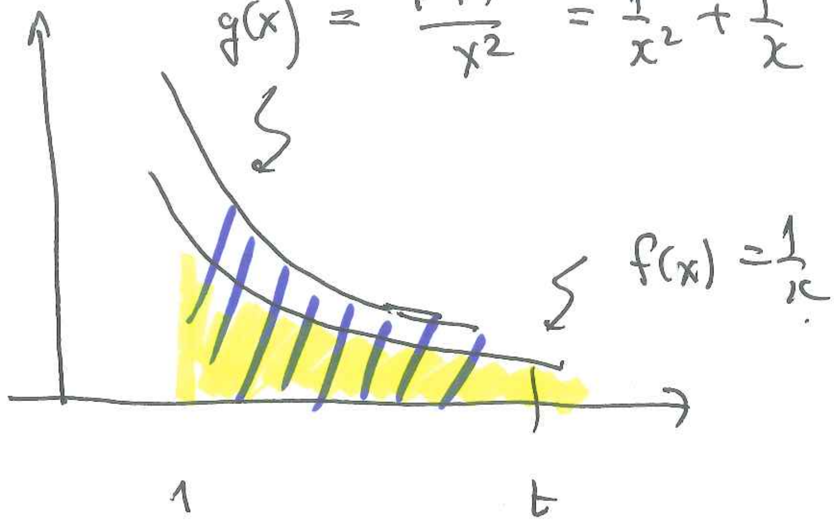
$$\lim_{t\to\infty} \int_1^t \frac{1}{x^p}\,dx = \frac{1}{p-1}$$

$p < 1$

$$\lim_{t\to\infty} \int_1^t \frac{1}{x^p}\,dx = +\infty$$

Puzzler #1

$$g(x) = \frac{1+x}{x^2} = \frac{1}{x^2} + \frac{1}{x}$$

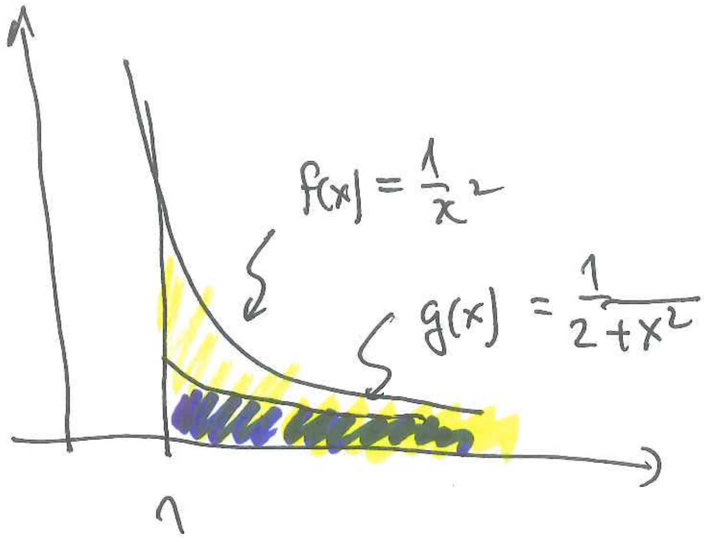


$$\frac{1+x}{x^2} \geq \frac{1}{x}$$

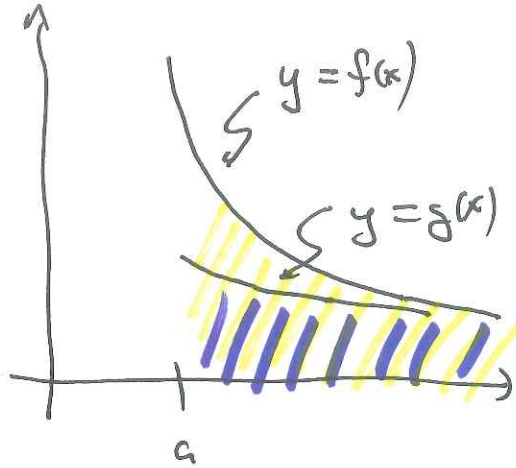
$$\int_1^t \frac{1+x}{x^2}\,dx \geq \int_1^t \frac{1}{x}\,dx$$

$$\lim_{t\to\infty} \int_1^t \frac{1+x}{x^2}\,dx \geq \lim_{t\to\infty} \int_1^t \frac{1}{x}\,dx = +\infty$$

Puzzler #2



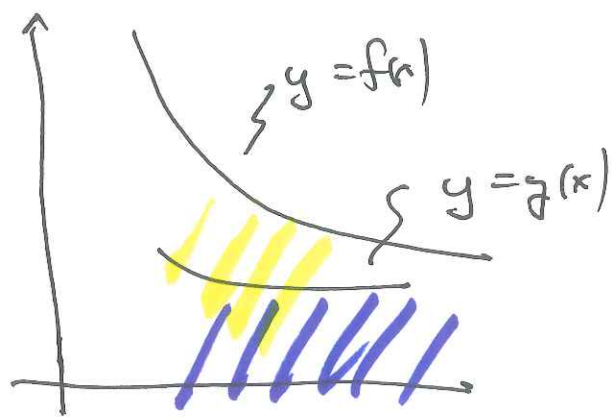
#1



$\int_a^\infty f(x)\,dx$ converges $\Rightarrow$

$\int_a^\infty g(x)\,dx$ "

#2



If $\int_a^\infty g(x)\,dx$ diverges,

$\int_a^\infty f(x)\,dx$ diverge.

Does $\displaystyle\int_1^\infty \frac{x+1}{\sqrt{x^5+1}}\,dx$ converge?

Rough + ready technique:

Look at leading powers of x in numerator + denominator

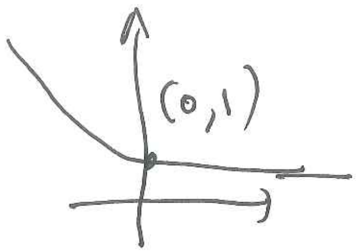
$$\frac{x+1}{\sqrt{x^5+1}} \approx \frac{x}{x^{5/2}} \simeq \frac{1}{x^{3/2}}$$

More precisely:

$$\boxed{\frac{x+1}{\sqrt{x^5+1}}} = \frac{x\left(1+\frac{1}{x}\right)}{\sqrt{x^5\left(1+\frac{1}{x^5}\right)}} = \frac{x\left(1+\frac{1}{x}\right)}{x^{5/2}\sqrt{1+\frac{1}{x^5}}}$$

$$= x^{-3/2}\left[\frac{1+\frac{1}{x}}{\sqrt{1+\frac{1}{x^5}}}\right] \leq \boxed{\frac{2}{1}\,x^{-3/2}}$$

$$e^{-x} \leq 1 \quad \text{if } x \geq 0$$



$$\int_a^{\infty} f(x)\,dx \quad \underline{\underline{\text{converges}}} \quad \text{if}$$

$$\lim_{t \to \infty} \left(\int_a^t f(x)\,dx \right) \quad \text{exists}$$

and $\underline{\underline{\text{diverges}}}$ otherwise

D $\displaystyle\int_1^{\infty} \frac{1}{1+x}\,dx = \lim_{t\to\infty} \int_1^{t} \frac{1}{1+x}\,dx$ $\quad u = 1+x$

$\displaystyle = \lim_{t\to\infty} \left(\int_2^{1+t} \frac{1}{u}\,du \right)$

$\dfrac{1}{x}$

$\displaystyle = \lim_{t\to\infty} \left\{ \left(\log(1+t) - \ln 2 \right) \right.$

C $\displaystyle\int_0^{\infty} \frac{3x}{x^2+4}\,dx$ $\qquad \dfrac{x}{x^2} \sim \dfrac{1}{x}$

B $\displaystyle\int \frac{4}{\sqrt{x}}\,dx = 4\int \frac{1}{x^{1/2}}\,dx$

DIV

A $\displaystyle \underbrace{\frac{1+e^{-x}}{x^2+4}} \leq \frac{2}{x^2+4} \leq \underbrace{\frac{2}{x^2}}$

Sequences

Formula $a_n = 2^{-n}$

Table

n	1	2	3	4
a_n	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{1}{8}$	$\frac{1}{16}$

List $\frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{16}, \frac{1}{32}, \ldots$

$a_n = 1 - \frac{1}{n}$

Table

n	1	2	3	4	5
a_n	0	$\frac{1}{2}$	$\frac{2}{3}$	$\frac{3}{4}$	$\frac{4}{5}$

$0, \frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \frac{4}{5}, \ldots$