MA 213 — Calculus III Fall 2017
Exam 1 September 20, 2017

Exam Scores

Do not write in the table below

Name: KEY

Section: ________

Last 4 digits of student ID #: ________

- No books or notes may be used.
- Turn off all your electronic devices and do not wear ear-plugs during the exam.
- You may use a calculator, but not one which has symbolic manipulation capabilities or a QWERTY keyboard.
- Additional blank sheets for scratch work are available upon request.
- All questions are free response questions. Show all your work on the page of the problem. Clearly indicate your answer and the reasoning used to arrive at that answer. Unsupported answers may not receive credit.

Question	Score	Total
1		9
2		8
3		8
4		9
5		8
6		10
7		9
8		9
9		10
10		10
11		10
Total		100

Free Response. Show your work!

1. (9 points) Find the center and radius of the sphere

$$2x^2 + 2y^2 + 2z^2 = 8x - 24z + 1.$$

$$2x^2 - 8x + 2y^2 + 2z^2 + 24z = 1$$

$$2(x^2 - 4x + 4) + 2y^2 + 2(z^2 + 12z + 36) = 1 + 8 + 72 = 81$$

$$(x-2)^2 + y^2 + (x+6)^2 = 81/2$$

$$\boxed{\text{Center: } (2, 0, -6) \qquad \text{Radius } 9/\sqrt{2}}$$

2. (8 points) Find a unit vector parallel to $\mathbf{a} = \langle 8, -1, 4 \rangle$ and having negative first coordinate.

A unit vector parallel to $\vec{a}$ is $\dfrac{\vec{a}}{|\vec{a}|}$

$$|\vec{a}| = \sqrt{8^2 + 1^2 + 16} = \sqrt{81} = 9$$

$$\frac{\vec{a}}{|\vec{a}|} = \left\langle \frac{8}{9}, \frac{-1}{9}, \frac{4}{9} \right\rangle$$ To make 1st coord. negative

multiply by -1 : $\boxed{-\dfrac{\vec{a}}{|\vec{a}|} = \left\langle -\dfrac{8}{9}, \dfrac{1}{9}, \dfrac{-4}{9} \right\rangle}$

3. (8 points) Find a vector that is orthogonal to both $\mathbf{i} + \mathbf{j}$ and $\mathbf{i} + \mathbf{k}$.

Use the cross product

$$(\hat{i} + \hat{j}) \times (\hat{i} + \hat{k}) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 1 & 0 \\ 1 & 0 & 1 \end{vmatrix} = \boxed{\hat{i} - \hat{j} - \hat{k}}$$

4. (9 points) Find the volume of the parallelepiped with adjacent edges PQ, PR, and PS, where
$$P = (-2, 1, 0), \quad Q = (2, 3, 2), \quad R = (5, 4, -1), \quad S = (3, 6, 1).$$

$\vec{PQ} = \langle 4, 2, 2 \rangle \quad \vec{PR} = \langle 7, 3, -1 \rangle \quad \vec{PS} = \langle 5, 5, 1 \rangle$

$$\vec{PQ} \cdot (\vec{PR} \times \vec{PS}) = \begin{vmatrix} 4 & 2 & 2 \\ 7 & 3 & -1 \\ 5 & 5 & 1 \end{vmatrix} = 4\begin{vmatrix} 3 & -1 \\ 5 & 1 \end{vmatrix} - 2\begin{vmatrix} 7 & -1 \\ 5 & 1 \end{vmatrix} + 2\begin{vmatrix} 7 & 3 \\ 5 & 5 \end{vmatrix}$$

$= 4 \cdot (3+5) - 2(7+5) + 2(35-15)$

72 $= 4 \cdot 8 - 2 \cdot 12 + 2 \cdot 20$

$\frac{24}{48}$ $= 32 - 24 + 40 = \boxed{48}$

5. (8 points) Find an equation of the plane through the point $(1, -1, -1)$ and parallel to the plane
$$5x - y - z = 6.$$

A normal for the plane $5x - y - z = 6$ is $\langle a, b, c \rangle = \langle 5, -1, -1 \rangle$

So a parallel plane has the equation

$5x - y - z = d$

To find d, set $(x, y, z) = (1, -1, -1)$:

$5(1) - (-1) - (-1) = \cancel{3}\ 7$

$\boxed{5x - y - z = \cancel{3}}\ 7$

6. (10 points) Find an equation of the plane with x-intercept a, y-intercept b, and z-intercept c.

$$Ax + By + Cz = D$$

$(a,0,0)$: $Aa = D$

$(0,b,0)$: $Bb = D$

$(0,0,c)$: $Cc = D$

Let's set $D = abc$ (there are other choices that are also ok)

$\therefore$ $A = bc$, $B = ac$, $C = ab$

$$\boxed{(bc)x + (ac)y + (ab)z = abc}$$

7. (9 points) Reduce the surface

$$9x^2 + 4z^2 = y^2 + 36$$

to one of the standard forms and classify it according to the provided table.

Divide by 36: (and move y^2 to the left)

$$\frac{x^2}{4} + \frac{z^2}{9} - \frac{y^2}{36} = 1$$

Consulting the table on p. 837 (which was given to students taking this exam) we see that $\frac{x^2}{4} + \frac{z^2}{9} - \frac{y^2}{36} = 1$ is a hyperboloid of <u>one</u> <u>sheet</u>

8. (9 points) Find a vector function that represents the curve of intersection of the hyperboloid $z = x^2 - y^2$ and the cylinder $x^2 + y^2 = 1$.

1) Parameterize $x = \cos t$, $y = \sin t$ to satisfy $x^2+y^2=1$

2) Solve for z: $z = x^2-y^2 = \cos^2 t - \sin^2 t$

$\therefore$ $\boxed{\vec{r}(t) = \langle \cos t, \sin t, \cos^2 t - \sin^2 t \rangle}$

9. (10 points) Find parametric equations for the tangent line to the curve

$$x = t\cos t, \quad y = t, \quad z = t\sin t$$

at the point $(-\pi, \pi, 0)$.

$\vec{r}(t) = \langle t\cos t, t, t\sin t \rangle$

$\vec{r}\,'(t) = \langle \cos t - t\sin t, 1, \sin t + t\cos t \rangle$

$\vec{r}(\pi) = \langle -\pi, \pi, 0 \rangle$ so use $t = \pi$

$\vec{r}\,'(\pi) = \langle \underset{a}{-1}, \underset{b}{1}, \underset{c}{-\pi} \rangle$ is tangent at $t = \pi$

Parametric eq'ns.

$$\boxed{\begin{aligned} x &= -\pi - t \\ y &= \pi + t \\ z &= 0 + -\pi t \end{aligned}}$$

10. (10 points) Compute the curvature $\kappa(t)$ of the plane curve

$$y = 2x - x^2.$$

$$\vec{r}(t) = \langle t, 2t - t^2, 0\rangle$$

$$\vec{r}\,'(t) = \langle 1, 2-2t, 0\rangle$$

$$\vec{r}\,''(t) = \langle 0, -2, 0\rangle$$

$$|\vec{r}\,'(t)| = \sqrt{1 + (2-2t)^2}$$

$$\vec{r}\,'(t) \times \vec{r}\,''(t) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2-2t & 0 \\ 0 & -2 & 0 \end{vmatrix} = (-2 - (2-2t))\,\hat{k} = (2t)\,\vec{k}$$

$$\kappa(t) = \frac{|\vec{r}\,'(t) \times \vec{r}\,''(t)|}{|\vec{r}\,'(t)|^3} = \frac{|2t|}{(1 + (2-2t)^2)^{3/2}}$$

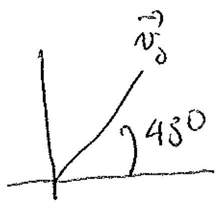
One can also use the formula

$$\kappa(x) = \frac{|f''(x)|}{[1 + f'(x)^2]^{3/2}}$$

with $f(x) = 2x - x^2$ to get the same answer.

Free Response. Show your work!

11. (10 points) A ball is thrown from the ground at an angle of 45° to the ground. If the ball lands 90 m away, what was the initial speed of the ball? [You may need to use the value $g = 9.8$ m/sec^2 for the acceleration due to gravity. The answer should be in m/sec.]



$\vec{r}'(0) = \langle v_0 \cdot \frac{\sqrt{2}}{2}, v_0 \frac{\sqrt{2}}{2} \rangle$

$x(t) = v_0 \cdot \frac{\sqrt{2}}{2} t \quad \cancel{\frac{1}{2} 9.8 \cdot t^2}$

$y(t) = v_0 \cdot \frac{\sqrt{2}}{2} t - \frac{1}{2} \cdot 9.8 \cdot t^2$

If t_i is the time of impact,

$x(t_i) = 90$ m

$\therefore \quad 90 \text{ m} = v_0 \frac{\sqrt{2}}{2} t_i \quad -(1)$

Also

$y(t_i) = 0$

$\therefore 0 = v_0 \frac{\sqrt{2}}{2} t_i - 4.9 t_i^2 = t_i (v_0 \frac{\sqrt{2}}{2} - 4.9 t_i)$

So $v_0 \frac{\sqrt{2}}{2} = 4.9 t_i \quad -(2)$

From (1), $90 \text{ m} = (4.9 t_i) \cdot t_i$

$t_i^2 = \frac{90 \text{ m}}{4.9 \text{ m/sec}^2} \Rightarrow t_i = 4.286$ sec

From (1) again

$90 \text{ m} = v_0 \frac{\sqrt{2}}{2} \cdot 4.286 \rightarrow v_0 = 29.7 \frac{\text{m}}{\text{sec}}$