Quiz 2

Name: ____________________ Section and/or TA: ____________

Answer all questions in a clear and concise manner. Unsupported answers will receive *no credit*.

1. (a) (4 points) Let P be the plane through the points $A(2,0,0)$, $B(-1,1,2)$ and $C(2,-2,2)$. Find a **unit** vector that is orthogonal to the plane P.

Solution: Let $\mathbf{u} = (-\mathbf{i}+\mathbf{j}+2\mathbf{k}) - (2\mathbf{i}) = -3\mathbf{i}+\mathbf{j}+2\mathbf{k}$ be the vector from A to B and $\mathbf{v} = (2\mathbf{i}-2\mathbf{j}+2\mathbf{k}) - (2\mathbf{i}) = -2\mathbf{j}+2\mathbf{k}$ be the vector from A to C. Then any vector orthogonal to both $\mathbf{u}$ and $\mathbf{v}$ is orthogonal to P. We can find one by computing $\mathbf{u}\times\mathbf{v}$.

$$\mathbf{u}\times\mathbf{v} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -3 & 1 & 2 \\ 0 & -2 & 2 \end{vmatrix} = \begin{vmatrix} 1 & 2 \\ -2 & 2 \end{vmatrix}\mathbf{i} - \begin{vmatrix} -3 & 2 \\ 0 & 2 \end{vmatrix}\mathbf{j} + \begin{vmatrix} -3 & 1 \\ 0 & -2 \end{vmatrix}\mathbf{k}$$

$$= 6\mathbf{i} + 6\mathbf{j} + 6\mathbf{k}$$

Thus, the two unit vectors orthogonal to P are $\frac{1}{\sqrt{3}}\mathbf{i} + \frac{1}{\sqrt{3}}\mathbf{j} + \frac{1}{\sqrt{3}}\mathbf{k}$ and $\frac{-1}{\sqrt{3}}\mathbf{i} + \frac{-1}{\sqrt{3}}\mathbf{j} + \frac{-1}{\sqrt{3}}\mathbf{k}$

(b) (1 point) Find the area of the of the triangle ΔABC

Solution: $|\mathbf{u}\times\mathbf{v}|$ gives us the area of the parallelogram whose adjacent sides are AB and AC. So, the are of the triangle ΔABC is

$$\frac{1}{2}|\mathbf{u}\times\mathbf{v}| = \frac{\sqrt{108}}{2} = \frac{\sqrt{4\cdot 27}}{2} = 3\sqrt{3}$$