

MA 213 Worksheet #9

Sections 14.1 and 14.2

9/25/18

1 14.1.9,11

(a) Let $g(x, y) = \cos(x + 2y)$.

i. Evaluate $g(2, 1)$.

ii. Find the domain of g .

iii. Find the range of g .

(b) Let $f(x, y, z) = \sqrt{x} + \sqrt{y} + \sqrt{z} + \ln(4 - x^2 - y^2 - z^2)$

i. Evaluate $f(1, 1, 1)$.

ii. Find and describe the domain of f .

2 14.1.45, 49 For each of the following functions, draw a contour map showing several level curves (then compare with a sketch or computer generated graph of the function).

(a) $f(x, y) = x^2 - y^2$

(b) $f(x, y) = ye^x$

3 14.1.55 A thin metal plate, located in the xy -plane, has temperature $T(x, y)$ at point (x, y) . Sketch some level curves (isothermals) if the temperature function is given by

$$T(x, y) = \frac{100}{1 + x^2 + 2y^2}$$

4 14.2.1 Suppose $\lim_{(x,y) \rightarrow (3,1)} f(x, y) = 6$. What can you say about the value of $f(3, 1)$? What if we also know that f is continuous?

5 14.2.5,9,11,13,16 For each of the following, find the limit if it exists. Otherwise show that the limit does not exist.

(a) $\lim_{(x,y) \rightarrow (3,2)} (x^2y^3 - 4y^2)$

(b) $\lim_{(x,y) \rightarrow (0,0)} \frac{x^4 - 4y^2}{x^2 + 2y^2}$

(c) $\lim_{(x,y) \rightarrow (0,0)} \frac{y^2 \sin^2(x)}{x^4 + y^4}$

(d) $\lim_{(x,y) \rightarrow (0,0)} \frac{xy}{\sqrt{x^2 + y^2}}$

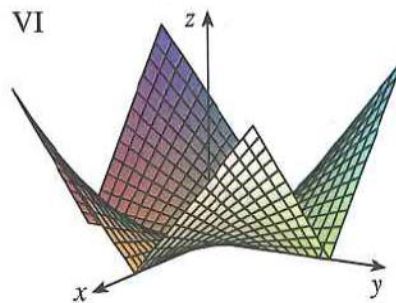
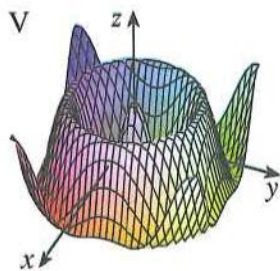
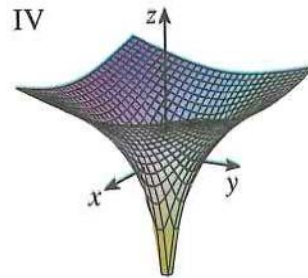
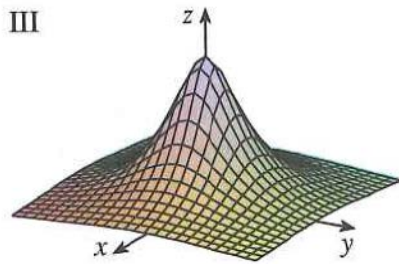
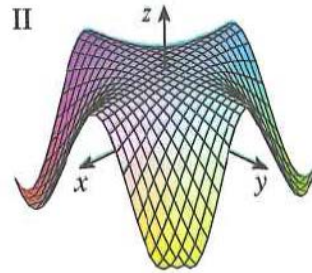
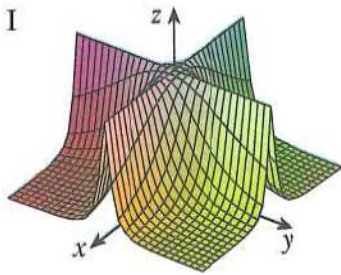
(e) $\lim_{(x,y) \rightarrow (0,0)} \frac{xy^4}{x^4 + y^4}$

6 14.2.31,35,37 Determine the set of points at which the function is continuous:

- (a) $F(x, y) = \frac{1+x^2+y^2}{1-x^2-y^2}$
 (b) $f(x, y, z) = \arcsin(x^2 + y^2 + z^2)$
 (c) $f(x, y) = \begin{cases} \frac{x^2 y^3}{2x^2 + y^2} & (x, y) \neq (0, 0) \\ 1 & (x, y) = (0, 0) \end{cases}$

7 14.1.32 Match the function with its graph, giving reasons for your choices.

- (a) $f(x, y) = \frac{1}{1+x^2+y^2}$
 (b) $f(x, y) = \frac{1}{1+x^2 y^2}$
 (c) $f(x, y) = \ln x^2 + y^2$
 (d) $f(x, y) = \cos \sqrt{x^2 + y^2}$
 (e) $f(x, y) = |xy|$
 (f) $f(x, y) = \cos xy$



8 14.1.61-66 Match the function with both its graph and its contour map, giving reasons for your choices.

- (a) $f(x, y) = \sin(xy)$
- (b) $f(x, y) = e^x \cos(y)$
- (c) $f(x, y) = \sin(x - y)$
- (d) $f(x, y) = \sin(x) - \sin(y)$
- (e) $f(x, y) = (1 - x^2)(1 - y^2)$
- (f) $f(x, y) = \frac{x-y}{1+x^2+y^2}$

