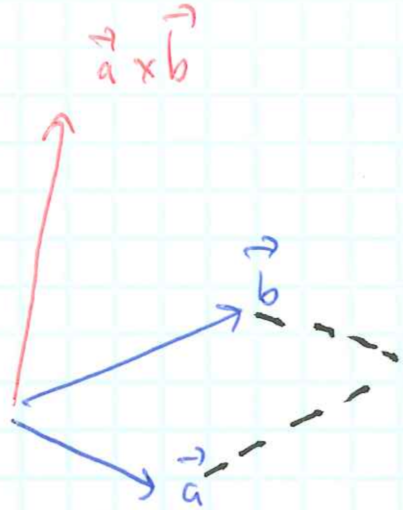
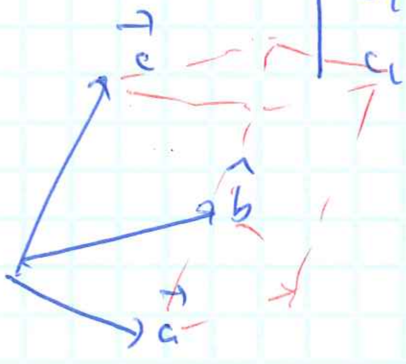
9/9/2019 ①

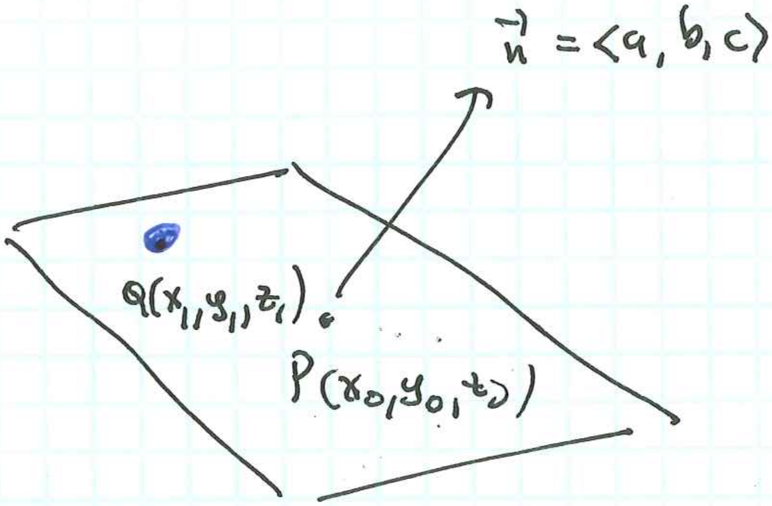


$|\vec{a} \times \vec{b}|$ = area of ~~parallelogr~~ parallelogram spanned by $\vec{a}$ and $\vec{b}$

$$\vec{a} \cdot (\vec{b} \times \vec{c}) = \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$$



9/9/2019 ②



Q lies in the plane if $\vec{PQ} \perp \hat{n}$

i.e., $\vec{n} \cdot \vec{PQ} = 0$

$$\langle a, b, c \rangle \cdot \langle x_1 - x_0, y_1 - y_0, z_1 - z_0 \rangle = 0$$

$$a(x_1 - x_0) + b(y_1 - y_0) + c(z_1 - z_0) = 0$$

$$a(x - x_0) + b(y - y_0) + c(z - z_0) = 0$$

OR: $ax + by + cz = d$

Ex: Find the equation of the plane through $(1, 1, 0)$ with normal $\langle 1, 2, 3 \rangle$

Sol'n: $1x + 2y + 3z = d$

To find d: $1 \cdot 1 + 2 \cdot 1 + 3 \cdot 0 = 1 + 2 = 3 = d$

9/19/19

Problem: $\langle a, b, c \rangle = \langle 1, -1, 2 \rangle$

$\langle x_0, y_0, z_0 \rangle =$

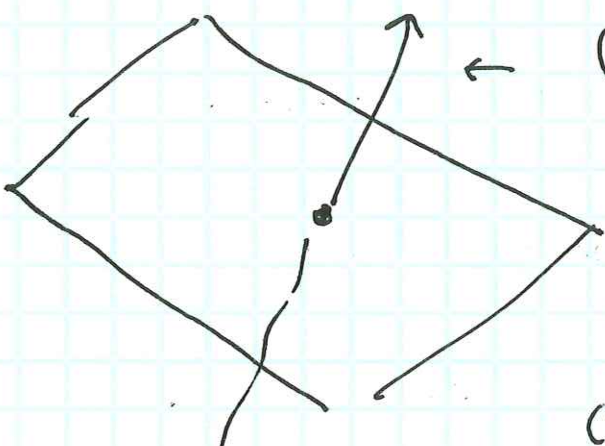
$d = ?$

$1x - 1y + 2z = d$

$1 \cdot 2 - 1 \cdot 2 + 2 \cdot 2 = d$

$d = 4$

$x - y + 2z = 4$



$\leftarrow \; (-7, 0, 0) + t\langle -7, 3, 3 \rangle$

$\vec{u} = \langle -7, 3, 3 \rangle$

$(x_0, y_0, z_0) = (0, 0, -7)$

$$ax + by + cz = d$$

Intersection w/ x-axis $(y = z = 0)$

$$ax + b\cdot 0 + c\cdot 0 = d$$

$$ax = d$$

$$x = \frac{d}{a}$$

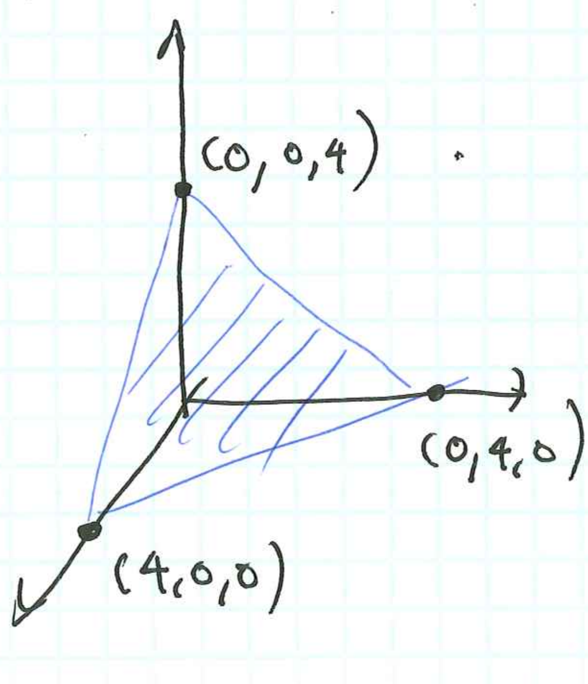
Intersection w/ y-axis: $(x = z = 0)$

$$a\cdot 0 + b\cdot y + c\cdot 0 = d$$

$$by = d$$

$$y = d/b$$

Ex: $x + y + z = 4$



$2x + y + 3z = 4$

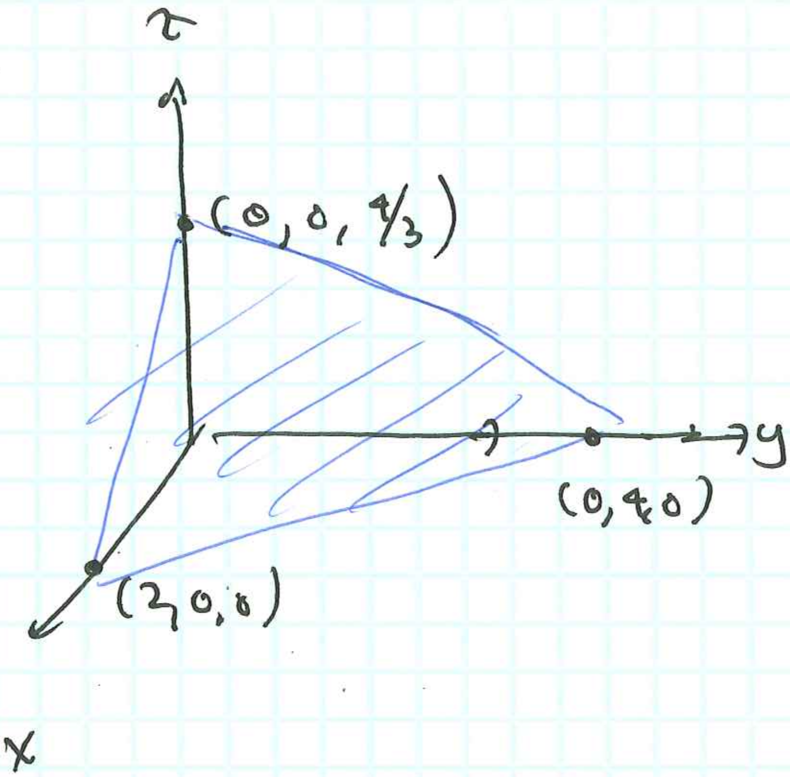
x-axis: $2x = 4$

$x = 2$

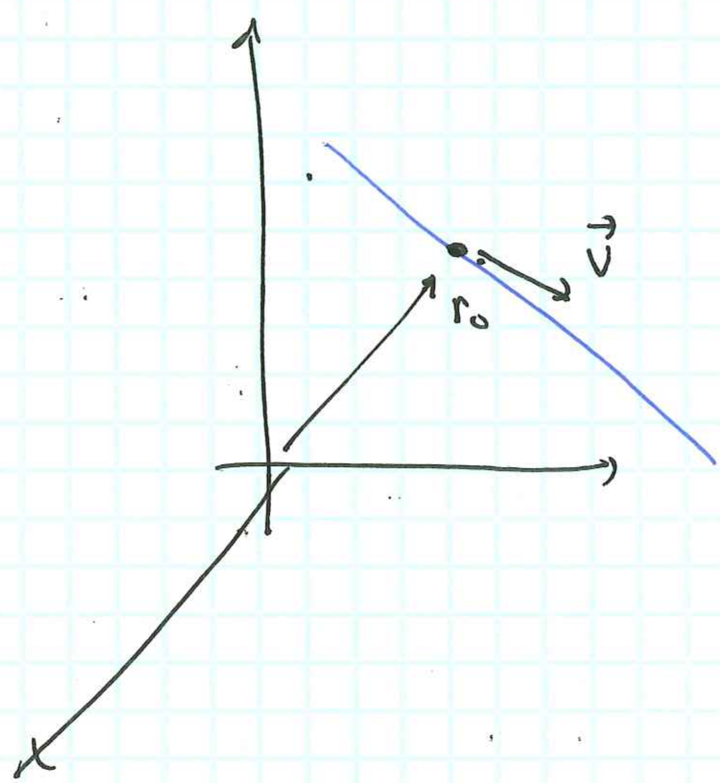
y-axis: $y = 4$

z-axis: $3z = 4$

$z = 4/3$



$\vec{r}(t) = \vec{r_0} + t\vec{v}$



L_1: $x = 2+s$

$y = 3-2s$

$z = 1-3s$

L_2: $x = 3+t$

$y = -4+3t$

$z = 2-7t$

$\vec{v}_1 = \langle 1, -2, -3 \rangle$

$\vec{v}_2 = \langle 1, 3, -7 \rangle$

Not parallel

Intersect? See if can find s, t so:

① $2+s = 3+t$

② $3-2s = -4+3t$

③ $1-3s = 2-7t$

From ①: $s = 1+t$

Subs in ② $3-2(1+t) = -4+3t$

$1-2t = -4+3t$

$5 = 5t$

$t = 1$

$s = 2$

Consistent w/ ③ ?

$1-3\cdot 2 = -5$

$2-7\cdot 1 = -5$ ✓

$\vec{n}_1 = \langle 1, 2, 3 \rangle$

$\vec{n}_2 = \langle 1, -1, 1 \rangle$

$$\vec{v} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 1 & 2 & 3 \\ 1 & -1 & 1 \end{vmatrix} = \hat{\imath}\,(5) + \hat{\jmath}\,(2) + \hat{k}\,(-3)$$

$$= 5\hat{\imath} + 2\hat{\jmath} - 3\hat{k}$$

$y = z = 0 \qquad x = 1$

$(1, 0, 0)$ belongs to both planes.

$\vec{r}(t) = \langle 1, 0, 0 \rangle + t\,\langle 5, 2, -3 \rangle$