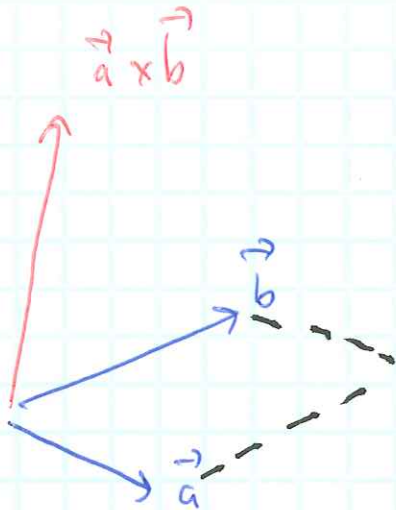


9/9/2019 ①

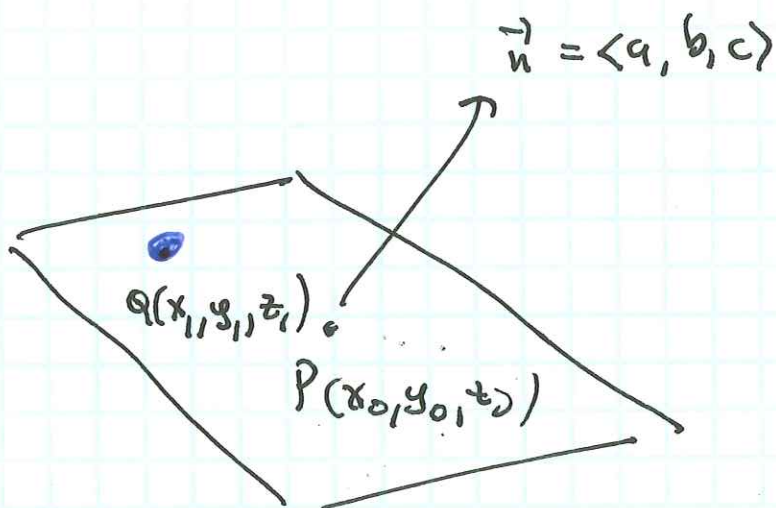


$|\vec{a} \times \vec{b}|$  = area of ~~parallelogr~~  
parallelogram spanned  
by  $\vec{a}$  and  $\vec{b}$

$$\vec{a} \cdot (\vec{b} \times \vec{c}) = \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$$

A diagram on grid paper showing three blue vectors,  $\vec{a}$ ,  $\vec{b}$ , and  $\vec{c}$ , originating from the same point. Red dashed lines and arrows project these vectors onto the x, y, and z axes, labeled  $a_1, a_2, a_3$ ,  $b_1, b_2, b_3$ , and  $c_1, c_2, c_3$  respectively.

9/9/2019 (2)



Q lies in the plane if  $\vec{PQ} \perp \hat{n}$

i.e.,  $\vec{n} \cdot \vec{PQ} = 0$

$$\langle \underline{a}, \underline{b}, \underline{c} \rangle \cdot \langle \underline{x_1} - \underline{x_0}, \underline{y_1} - \underline{y_0}, \underline{z_1} - \underline{z_0} \rangle = 0$$

$$a(x_1 - x_0) + b(y_1 - y_0) + c(z_1 - z_0) = 0$$

$$a(x - x_0) + b(y - y_0) + c(z - z_0) = 0$$

or:  $ax + by + cz = d$

Ex: Find the equation of the plane through  $(\underline{1}, \underline{1}, \underline{0})$  with normal  $\langle \underline{1}, \underline{2}, \underline{3} \rangle$

Sol'n:  $1x + 2y + 3z = d$

To find  $d$ :  $1 \cdot \underline{1} + 2 \cdot \underline{1} + 3 \cdot \underline{0} = 1 + 2 = 3 = d$

9/09/19

②

Problem:  $\langle a, b, c \rangle = \langle 1, -1, 2 \rangle$

$$\langle x_0, y_0, z_0 \rangle =$$

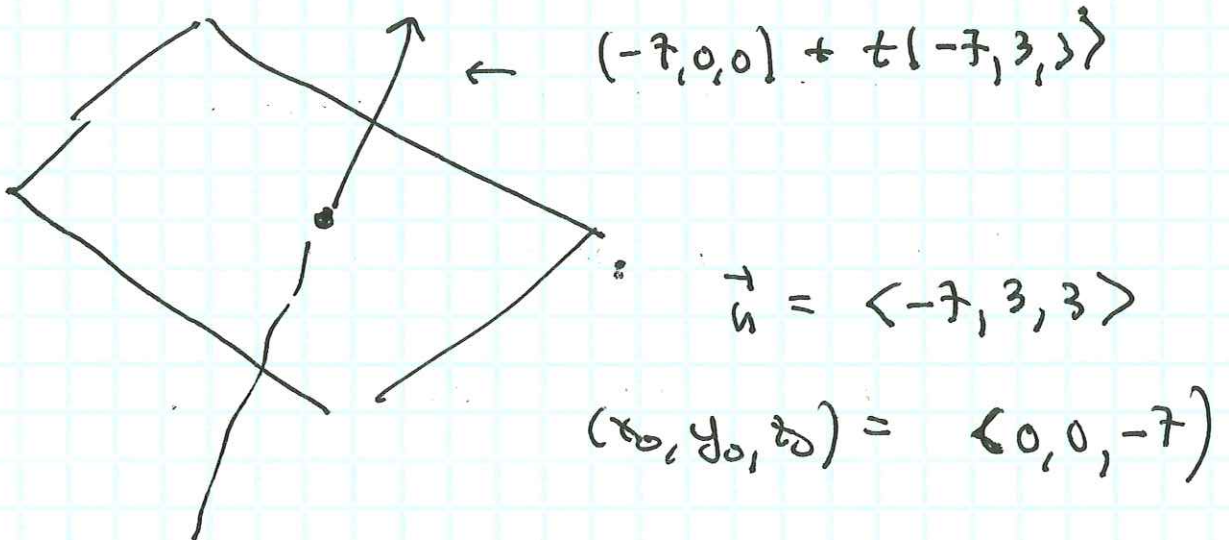
$$d = ?$$

$$1x - 1y + 2z = d$$

$$1 \cdot 2 - 1 \cdot 2 + 2 \cdot 2 = d$$

$$d = 4$$

$$x - y + 2z = 4$$



4

$$ax + by + cz = d$$

Intersection w/ x-axis ( $y = z = 0$ )

$$ax + b \cdot 0 + c \cdot 0 = d$$

$$ax = d$$

$$x = \frac{d}{a}$$

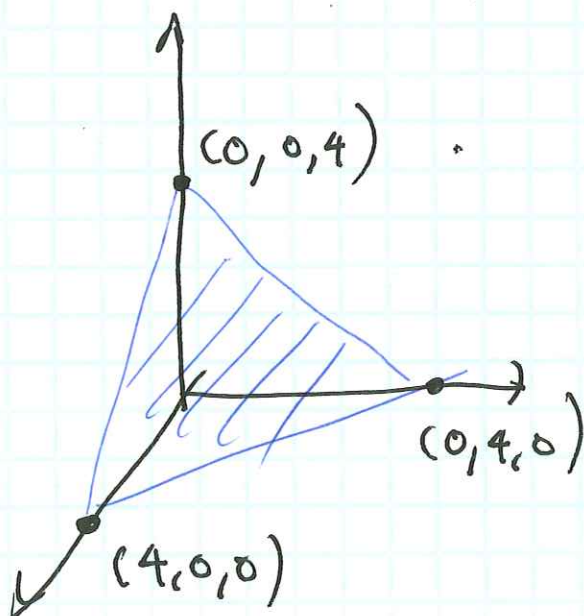
Intersection w/ y-axis: ( $x = z = 0$ )

$$a \cdot 0 + b \cdot y + c \cdot 0 = d$$

$$by = d$$

$$y = d/b$$

Ex:  $x + y + z = 4$





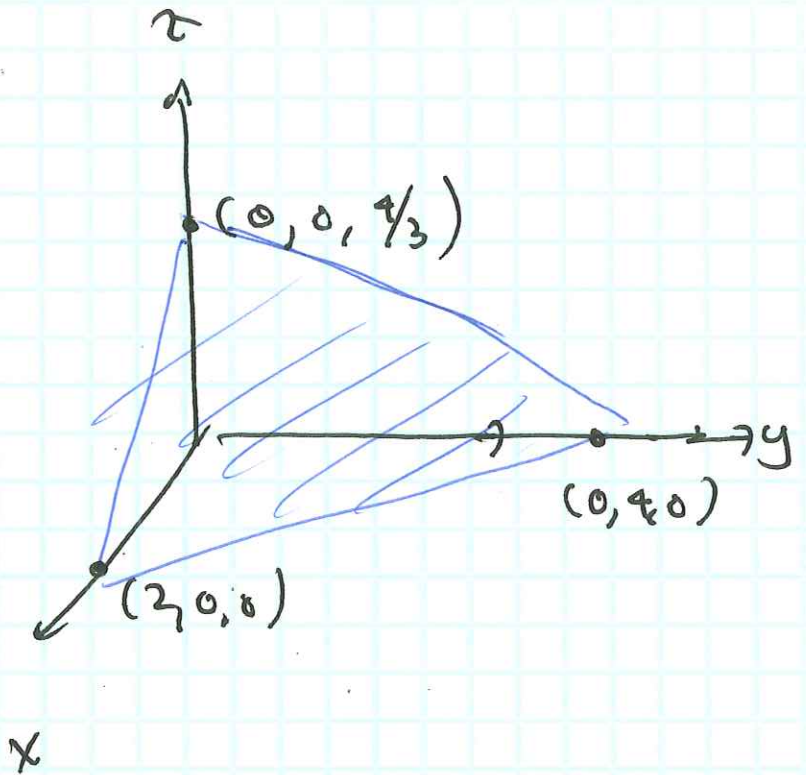
(5)

$$2x + y + 3z = 4$$

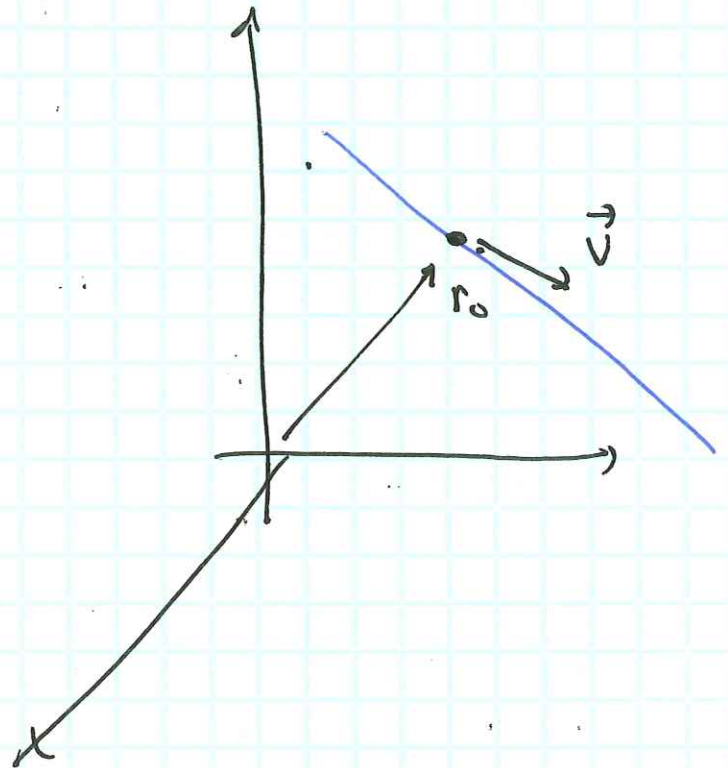
x-axis:  $2x = 4$   
 $x = 2$

y-axis:  $y = 4$

z-axis:  $3z = 4$   
 $z = 4/3$



$$\vec{r}(t) = \vec{r}_0 + t\vec{v}$$



6

$$L_1: \begin{aligned} x &= 2 + s \\ y &= 3 - 2s \\ z &= 1 - 3s \end{aligned}$$

$$L_2: \begin{aligned} x &= 3 + t \\ y &= -4 + 3t \\ z &= 2 - 7t \end{aligned}$$

$$\vec{v}_1 = \langle 1, -2, -3 \rangle$$

$$\vec{v}_2 = \langle 1, 3, -7 \rangle$$

Not parallel

Intersect? See if can find  $s, t$  so:

$$(1) \quad 2 + s = 3 + t$$

$$(2) \quad \frac{3 - 2s}{1 - 3s} = \frac{-4 + 3t}{2 - 7t}$$

$$(3) \quad 1 - 3s = 2 - 7t$$

From (1):  $s = 1 + t$

Subs in (2):  $3 - 2(1+t) = -4 + 3t$

Consistent w/ (3)?

$$1 - 3 \cdot 2 = -5$$

$$2 - 7 \cdot 1 = -5$$

(✓)

$$1 - 2t = -4 + 3t$$

$$s = 5t$$

$$t = 1$$

$$s = 2$$

7

$$\vec{n}_1 = \langle 1, 2, 3 \rangle$$

$$\vec{n}_2 = \langle 1, 4, 1 \rangle$$

$$\begin{aligned} \vec{c} &= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & 3 \\ 1 & -1 & 1 \end{vmatrix} = \hat{i}(5) + \hat{j}(2) + \hat{k}(-3) \\ &= 5\hat{i} + 2\hat{j} - 3\hat{k} \end{aligned}$$

---

$$y = z = 0 \quad x = 1$$

$(1, 0, 0)$  belongs to both planes.

$$\vec{r}(t) = \langle t, 0, 0 \rangle + t \langle 5, 2, -3 \rangle$$