Math 213 - Exam I Review

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Reminders

- Access your WebWork account *only through Canvas*!
- Homework A5 on section 13.1-13.2 is due tonight!
- Exam 1 takes place tonight at 5 PM. Section 17 will meet in CB 118, and sections 18 and 19 will meet in CB 122.

Unit I: Geometry and Motion in Space

Learning Goals

- Find out how to ace Exam I

Dot Products and Cross Products

	Formula	Type	Geometry	Zero if...
Dot	$\mathbf{a} \cdot \mathbf{b}$	Scalar	Projections	$\mathbf{a}, \mathbf{b}$ orthogonal
Cross	$\mathbf{a} \times \mathbf{b}$	Vector	Area of a Parallelogram	$\mathbf{a}, \mathbf{b}$ parallel
Triple	$\mathbf{a} \cdot (\mathbf{b} \times \mathbf{c})$	Scalar	Volume of a Parallelepiped	$\mathbf{a}, \mathbf{b}, \mathbf{c}$ coplanar

Lines and Planes

Equation of a Line

A line is specified by a point $P(x_0, y_0, z_0)$ that it contains and a vector $\mathbf{v} = \langle a, b, c\rangle$ that points along it

Parametric: $x(t) = x_0 + at, \quad y(t) = y_0 + bt, \quad z(t) = z_0 + ct$

Symmetric: $\dfrac{x - x_0}{a} = \dfrac{y - y_0}{b} = \dfrac{z - z_0}{c}$

Equation of a Plane

A plane is specified by a point $P(x_0, y_0, z_0)$ in the plane and a vector $\mathbf{n} = \langle a, b, c\rangle$ that is normal to the plane

$$ax + by + cz = d$$

where a, b, c are the components of $\mathbf{n}$ and d is determined by substituting in (x_0, y_0, z_0)

Quadric Surfaces

Ellipsoid: $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$

Elliptic Paraboloid: $z = \frac{x^2}{a^2} + \frac{y^2}{b^2}$

Cone: $\frac{x^2}{a^2} + \frac{y^2}{b^2} = z^2$

Hyperboloid of One Sheet: $\frac{x^2}{a^2} - \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$

Saddle: $z = \frac{x^2}{a^2} - \frac{y^2}{b^2}$

You may also see similar equations but with x, y, z permuted or with the origin shifted

Know how to use the *method of traces* to identify a surface

Vector Functions and Space Curves

Vector Functions and Space Curves

A *vector function* takes the form

$$\mathbf{r}(t) = \langle f(t), g(t) \rangle \quad \text{or} \quad \mathbf{r}(t) = \langle f(t), g(t), h(t) \rangle$$

A *space curve* is the path in two- or three-dimensional space traced out by a vector function $\mathbf{r}(t)$ for a certain range of t

The *tangent vector* to a space curve is the vector

$$\mathbf{r}'(t) = \langle f'(t), g'(t) \rangle \quad \text{or} \quad \mathbf{r}'(t) = \langle f'(t), g'(t), h'(t) \rangle$$

The tangent vector $\mathbf{r}'(a)$ to a space curve gives its instantaneous rate of change at $t = a$
Its magnitude $|\mathbf{r}'(a)|$ is the instantaneous speed of the space curve at $t = a$

To find the angle of intersection of two curves described by $\mathbf{r}_1(t)$ and $\mathbf{r}_2(t)$

- Find the time of intersection $t = a$
- Find the angle between the tangent vectors $\mathbf{r}_1'(a)$ and $\mathbf{r}_2'(a)$

Vector Algebra

If $|\mathbf{a}| = 3$ and $|\mathbf{b}| = 6$ what is $|\mathbf{a} + \mathbf{b}|$?

If a river 50m wide flows at 4 m/sec and a swimmer swims straight from one bank toward the other at 5 m/sec, how far downstream will the swimmer end up? What distance will she/he swim?

Determine whether the triangle with vertices $P(1, 0, 1)$, $Q(3, 0, 3)$, $R(-1, 0, 1)$ is isosceles.

Determine whether the points $P(1, 1, 0)$, $Q(3, 1, 2)$ and $R(4, 2, 2)$ are coplanar.

Quadric Surfaces

Identify the surface

$$z = x^2 + 2x - y^2 + 2y$$

Identify the surface

$$z^2 - 2z + x^2 + y^2 - 4y = 1$$

Identify the surface

$$y^2 = x^2 + z^2$$

Identify the surface

$$x^2 = y$$

Identify the surface

$$4x^2 + 9y^2 + z^2 = 36$$

Lines and Planes

Find the equation of a line segment from $P(1,-2,4)$ to $Q(3,2,3)$.

Determine whether the lines $\mathbf{r}_1(t) = \langle 1, 2+2t, 2+3t \rangle$ and $\mathbf{r}_2(t) = \langle 3, 2+5t, 7-t \rangle$ are parallel, intersecting, or skew.

Find the equation of a plane containing the three points

$$P(1,0,-1), \quad Q(2,3,4), \quad R(3,2,1).$$

Find the parametric equation of line of intersection between the planes

$$x - y + z = 5$$

and

$$2x + y + z = 6$$

Space Curves

Do the curves

$$\mathbf{r}_1(s) = \langle s, 2s-1, s^2 \rangle$$

and

$$\mathbf{r}_2(t) = \langle t-1, t^2-3, (t-1)^3 \rangle$$

intersect? Do they collide?

Find the point where the curves

$$\mathbf{r}_1(t) = \langle \cos t, \sin t, t \rangle$$

and

$$\mathbf{r}_2(t) = \langle t+1, t^2, t^3 \rangle$$

intersect, and angle of intersection between the curves.

Open Mike

Your questions?

Reminders

- Your exam is 5 PM to 7 PM. There will be 10 multiple choice questions and 4 free-response questions.
- If you are in section 17, you should go to CB 118. If you are in sections 18 or 19 you should go to CB 122. Please arrive at least 10 minutes before the start of the exam and please have your student ID with you. Bring a one-page notebook-paper-sized sheet of formulas, notes, etc., if you wish.
- Raw scores should be posted in Canvas by late Thursday. We'll announce in Friday's class whether there will be a curve.
- You will receive your exam papers back in Tuesday's recitation.
- If you have any concerns about the grading on your exam, please turn in your exams with an explanatory note by the end of Tuesday's recitation. Regrading requests submitted after Tuesday recitation will not be considered.

Good Luck!