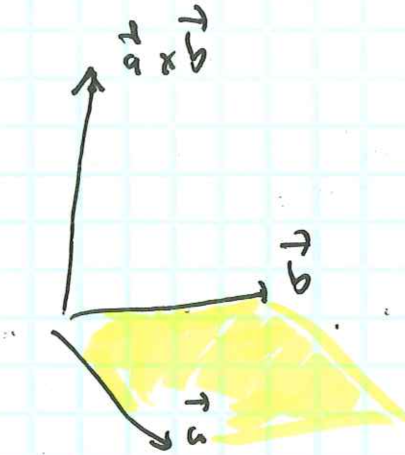
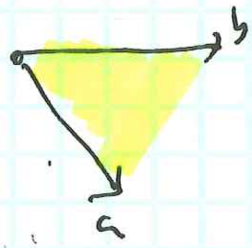
Cross

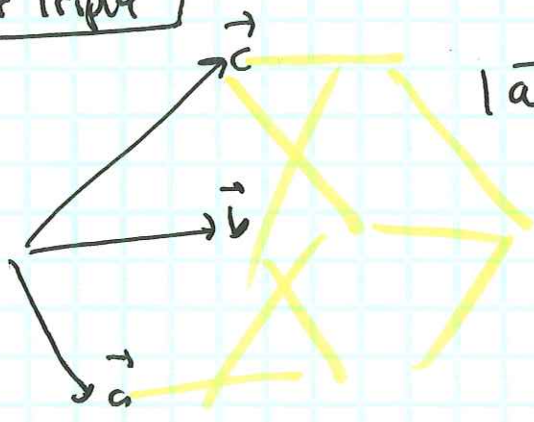


Direction: RHR

Magnitude:



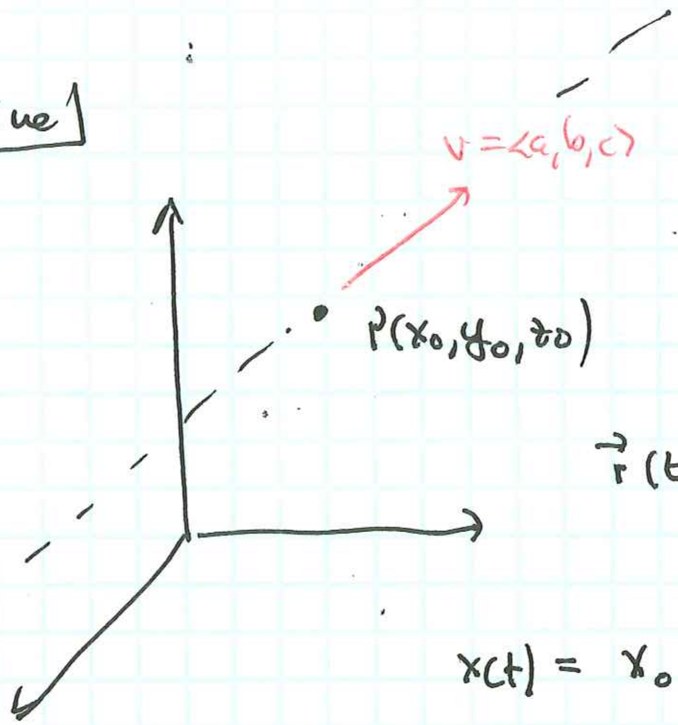
Scalar Triple



$|\vec{a}\cdot(\vec{b}\times\vec{c})|$ = volume of parallelepiped formed by $\vec{a}, \vec{b}, \vec{c}$

9/18/2019 ②

Line



$\vec{r}(t) = \langle x_0, y_0, z_0 \rangle + t\vec{v}$

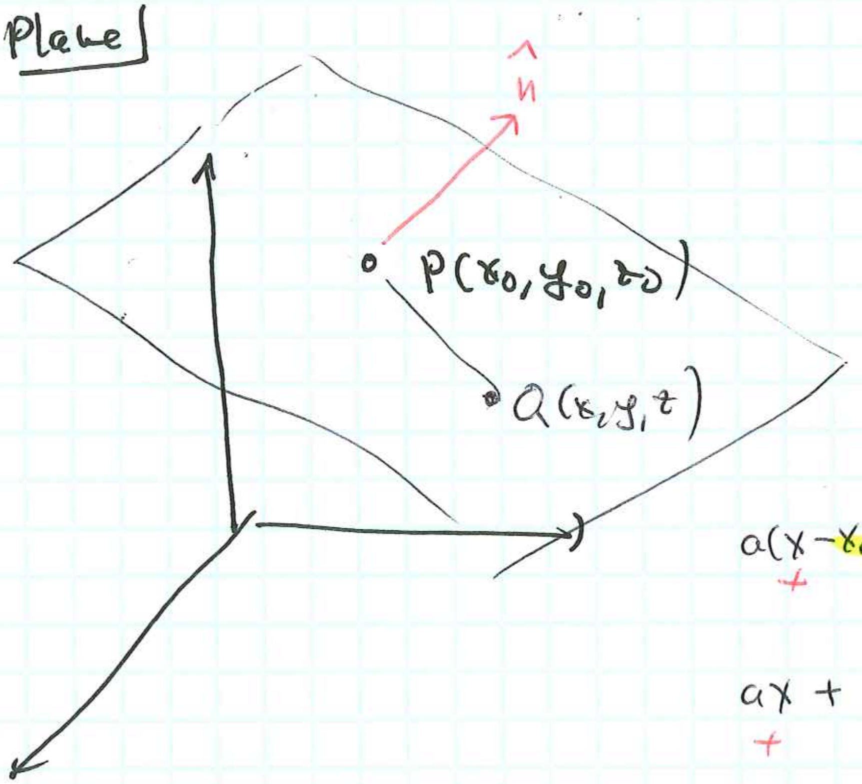
$= \langle x_0, y_0, z_0 \rangle + t\langle a, b, c \rangle$

$x(t) = x_0 + at$

$y(t) = y_0 + bt$

$z(t) = z_0 + ct$

Plane



$\vec{n} \cdot \vec{PQ} = 0$

$\langle a, b, c \rangle \cdot$

$\langle x - x_0, y - y_0, z - z_0 \rangle$

$= 0$

$a(x - x_0) + b(y - y_0) + c(z - z_0) = 0$

$ax + by + cz = d$

9/18/2019 3

Surfaces

$$z = \frac{x^2}{a^2} + \frac{y^2}{b^2}$$

trace in x,y plane $z=0$

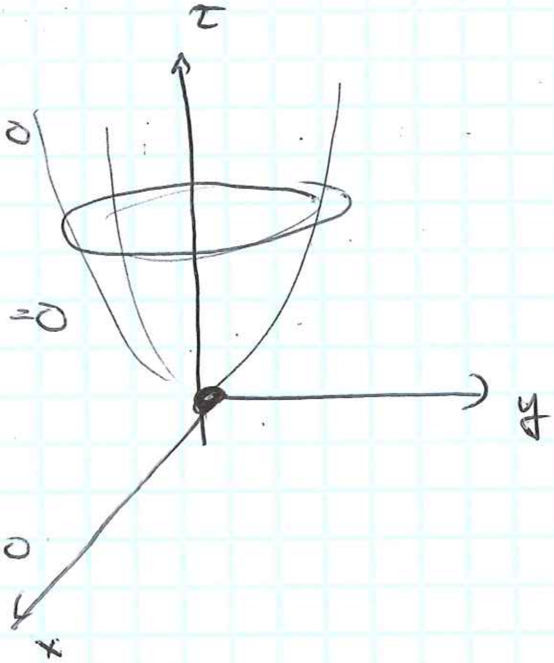
$z=0 \quad x=y=0$

" " yz plane $x=0$

$$z = \frac{y^2}{b^2}$$

" " xz plane $y=0$

$$z = \frac{x^2}{a^2}$$



Trace $z = h$

$$h = \frac{x^2}{a^2} + \frac{y^2}{b^2}$$

$$2x^2 + 4x + 4y^2 - 8y + z^2 = 0$$

$$2(x^2+2x) + 4(y^2-2y) + z^2 = 0$$

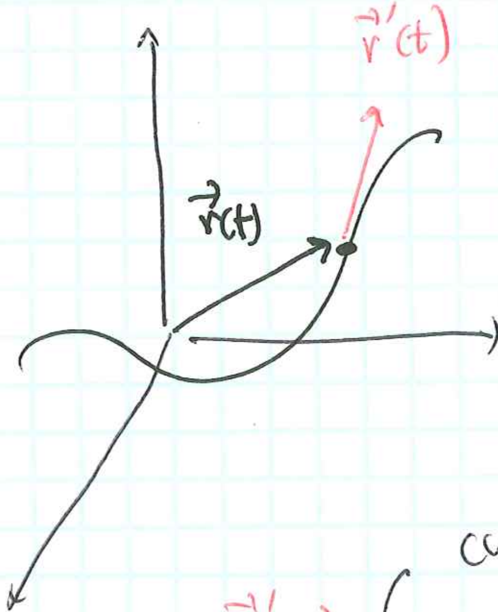
$$x^2 + 4x + 4 + y^2 - 8y + 16 + z^2 = 4 + 16$$

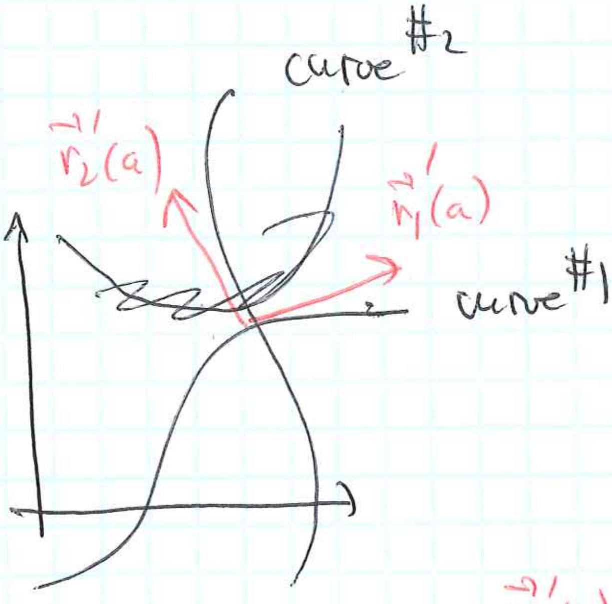
$$(x+2)^2 + (y-4)^2 + z^2 = 20$$

sphere w/ center $(-2, 4, 0)$

$$2(x^2+2x+1) + 4(y^2-2y+1) + z^2 = 2+4$$

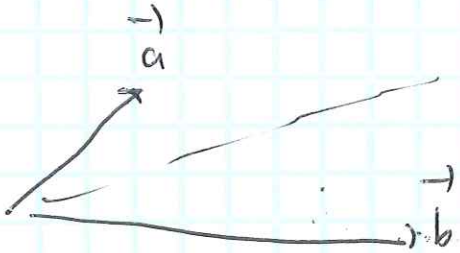
Space curves

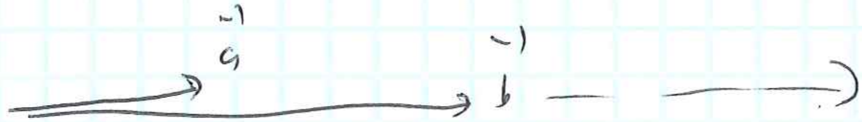


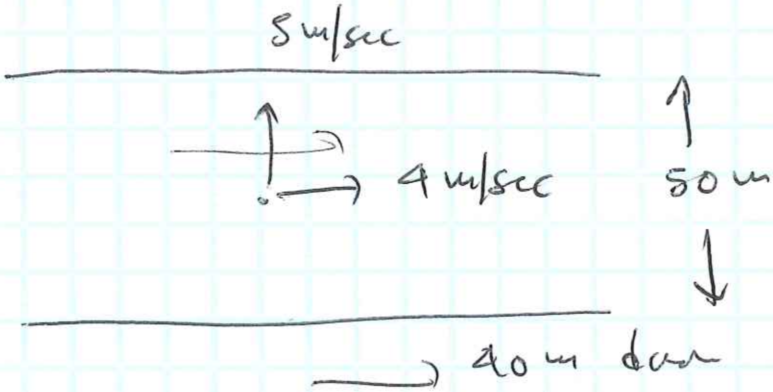


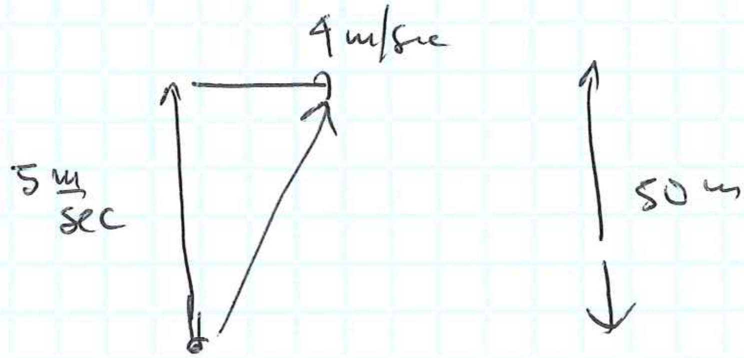
$$\cos\theta = \frac{\vec{r}_1'(a)\cdot\vec{r}_2'(a)}{|\vec{r}_1'(a)|\,|\vec{r}_2'(a)|}$$

$\boxed{\text{angle of intersection}}$

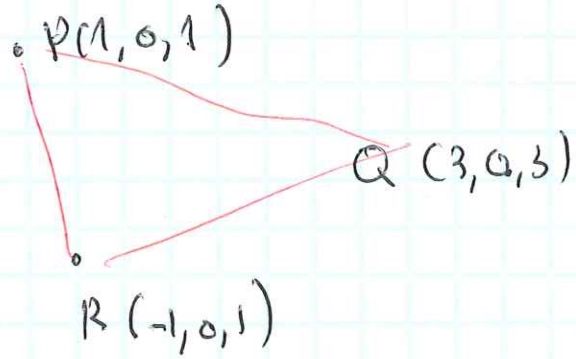








10sec swim



$\overrightarrow{PQ} = \langle 2, 0, 2 \rangle$ $\quad |\overrightarrow{PQ}| = \sqrt{8} = 2\sqrt{2}$

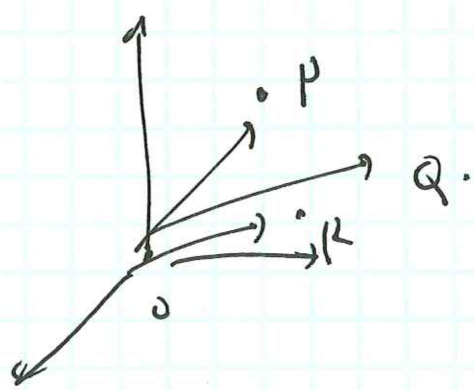
$\overrightarrow{QR} = \langle -4, 0, 2 \rangle$ $\quad |\overrightarrow{QR}| = \sqrt{16+4} = \sqrt{20}$

$\overrightarrow{RP} = \langle 2, 0, 0 \rangle$ $\quad |\overrightarrow{RP}| = 2$

P(1,1,0)

Q(3,1,2)

R(4,2,2)



$\vec{a} = \langle 1, 1, 0 \rangle$

$\vec{b} = \langle 3, 1, 2 \rangle$

$\vec{c} = \langle 4, 2, 2 \rangle$

$\vec{a} \cdot (\vec{b} \times \vec{c}) =$

$$\begin{vmatrix} 1 & 1 & 0 \\ 3 & 1 & 2 \\ 4 & 2 & 2 \end{vmatrix} = -2 + (-1)(-2)$$

$$= 0$$

$$z^2 - 2z + x^2 + y^2 - 4y = 1$$

$$z^2 - 2z + 1 + x^2 + y^2 - 4y + 4 = 1 + 1 + 4$$

$$(z-1)^2 + x^2 + (y-2)^2 = 6$$

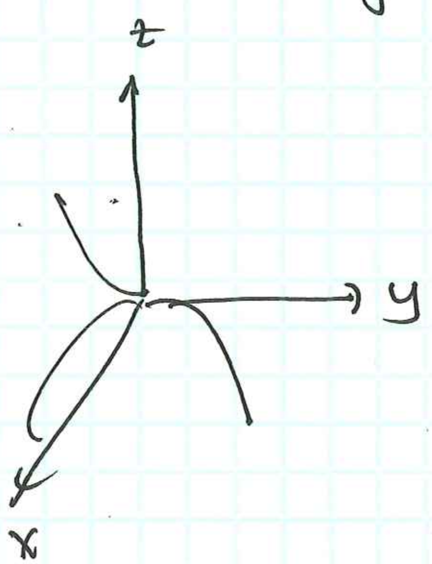
$$z = x^2 + 2x - y^2 + 2y$$

$$+1 \quad 1 + z = x^2 + 2x + 1 - (y^2 - 2y + 1)$$

$$z = (x+1)^2 - (y-1)^2$$

Saddle w/ center $(-1, 1, 0)$

Model: $z = x^2 - y^2$



Traces:

$x = 0$:

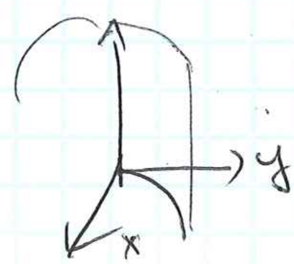
$z = -y^2$

$y = 0$:

$z = x^2$

---

8. $x^2 = y$



$P(1, -2, 4)$ to $Q(3, 2, 3)$

$\vec{PQ} = \langle 2, 4, -1 \rangle$

$\vec{r}(t) = \langle 1, -2, 4 \rangle + t\langle 2, 4, -1 \rangle$

$t = 0 \Leftrightarrow P$

$t = 1 \Leftrightarrow Q$

$0 \le t \le 1$

$P(1, 0, -1)$

$Q(2, 3, 4)$

$R(3, 2, 1)$

$\vec{PQ} = \langle 1, 3, 5 \rangle$

$\vec{PR} = \langle 2, 2, 2 \rangle$

$$\vec{n} = \vec{PQ} \times \vec{PR} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 3 & 5 \\ 2 & 2 & 2 \end{vmatrix}$$

$$= -4\hat{i} + 8\hat{j} - 4\hat{k}$$

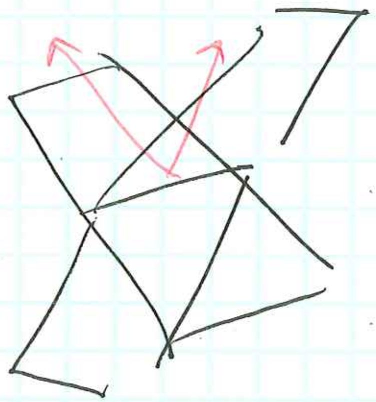
a b c

$-4x + 8y - 4z = \boxed{\cancel{0}\ 0}$

1 0 -1

$\cancel{2 \quad 2 \quad 2}$

$\cancel{-8 + 16 - 8}$



$\vec{n}_1 = \langle 1, -1, 1 \rangle$

$\vec{n}_2 = \langle 2, 1, 1 \rangle$

Find normal:
$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -1 & 1 \\ 2 & 1 & 1 \end{vmatrix}$$

$x - y + z = 5$

$2x + y + z = 6$

pt. on line:

set $x = 0$

$x = 0$: $-y + z = 5$

$y + z = 6$

$2z = 11$

$z = 11/2$

solve y:

Intersects

(1) $s = t - 1$

(2) $2s - 1 = t^2 - 3$

(3) $s^2 = (t-1)^3$

$\Rightarrow$

(1)' $t = 1 + s$

(2)' $2s - 1 = (1+s)^2 - 3$

$\cancel{2s} - 1 = s^2 + \cancel{2s} + 1 - 3$

$0 = s^2 + 2 - 3$

$0 = s^2 - 1$

$s = \pm 1$

Two possibilities:

$s = +1$ $\quad$ $s = -1$

$t = 2$ $\quad$ $t = 0$

(1) $s = +1$ $\quad$ $t = 2$

In eq'n (3): $1^2 = (2-1)^3$ ✓

(2) $s = -1$ $\quad$ $t = 0$

In eq'n (3): $s^2 = 1$ $\quad$ X

$(t-1)^3 = (-1)^3 = -1$