Arc Length

If $\vec{r}(t)$ is vector function and $a \le t \le b$,

the length of the space curve described by $\vec{r}(t)$, $a \le t \le b$,

$$L = \int_a^b |\vec{r}\,'(t)|\, dt$$

ws #1 (a) $\vec{r}(t) = \langle t, 3\cos(t), 3\sin(t) \rangle \quad -5 \le t \le 5$

~~$|\vec{r}\,'(t)|$:~~

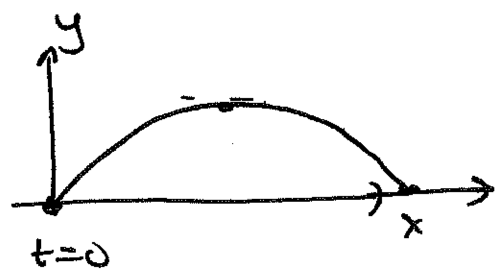
$$\vec{r}\,'(t) = \langle 1, -3\sin(t), 3\cos(t) \rangle$$

$$|\vec{r}\,'(t)| = \sqrt{1 + (-3\sin(t))^2 + (3\cos(t))^2}$$

$$= \sqrt{1+9}$$

$$= \sqrt{10}$$

$$\therefore L = \int_{-5}^{5} \sqrt{10}\, dt = 10\sqrt{10}$$



$$\vec{r}(t) = \langle 32t,\ 32t - 16t^2 \rangle$$

$$\vec{r}\,'(t) = \langle 32,\ 32 - 32t \rangle$$

$$\vec{r}\,''(t) = \langle 0,\ -32 \rangle$$

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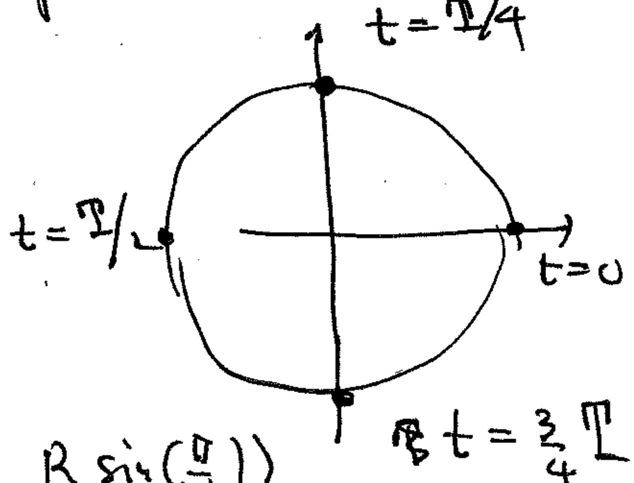
$$\vec{r}(t) = \langle R\cos(2\pi t/T),\ R\sin(2\pi t/T) \rangle$$

- Motion in a circle of radius R
- T is the period



$$\vec{r}(0) = \langle R, 0 \rangle$$

$$\cancel{\vec{r}\left(\tfrac{\pi}{2}\right) =}$$

$$\vec{r}\left(\tfrac{T}{4}\right) = \cancel{\vec{r}\left(\tfrac{\pi}{2}\right)}$$

$$\langle R\cos\left(\tfrac{\pi}{2}\right),\ R\sin\left(\tfrac{\pi}{2}\right) \rangle$$

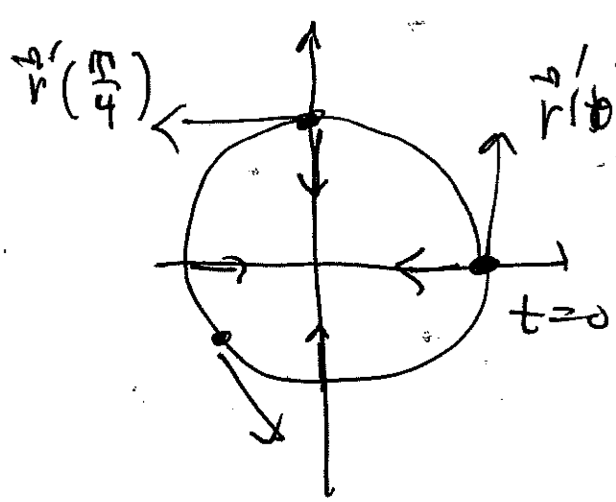
$$\vec{r}(t) = \left\langle R\cos\left(\frac{2\pi t}{T}\right),\ R\sin\left(\frac{2\pi t}{T}\right)\right\rangle$$

$$\vec{r}\,'(t) = \left\langle -\underline{\frac{2\pi}{T}}\cdot R\sin\left(\frac{2\pi t}{T}\right),\ \underline{\frac{2\pi}{T}}\cdot R\overset{\cos}{\cancel{\sin}}\left(\frac{2\pi t}{T}\right)\right\rangle$$

$$= \frac{2\pi R}{T}\left\langle -\sin\left(\frac{2\pi t}{T}\right),\ \cos\left(\frac{2\pi t}{T}\right)\right\rangle$$

$$\vec{r}\,''(t) = \frac{2\pi R}{T}\left\langle -\frac{2\pi}{T}\cos\left(\frac{2\pi t}{T}\right),\ -\frac{2\pi}{T}\sin\left(\frac{2\pi t}{T}\right)\right\rangle$$

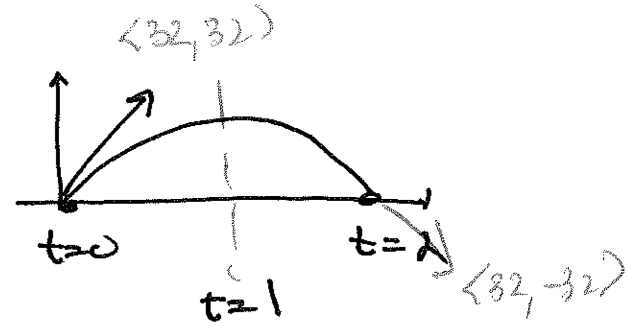
$$= -\frac{4\pi^2 R}{T^2}\left\langle \cos\left(\frac{2\pi t}{T}\right),\ -\sin\left(\frac{2\pi t}{T}\right)\right\rangle$$



$$\vec{r}(t) = \langle \underset{x(t)}{32t}, \underbrace{32t - 16t^2}_{y(t)} \rangle$$

$$\vec{r}''(t) = \langle 0, -32 \rangle$$

$$\vec{r}'(0) = \langle 32, 32 \rangle$$



Hits ground: $32t - 16t^2 = 0$

$$16t(2-t) = 0$$

$$t = 0 \quad \text{and} \quad t = 2$$

Speed when hits:

$$\vec{r}'(t) = \langle 32, 32 - 32t \rangle$$

$$\vec{r}'(2) = \langle 32, -32 \rangle$$

$$|\vec{r}'(2)| = \sqrt{32^2 + 32^2} = 32\sqrt{2}$$

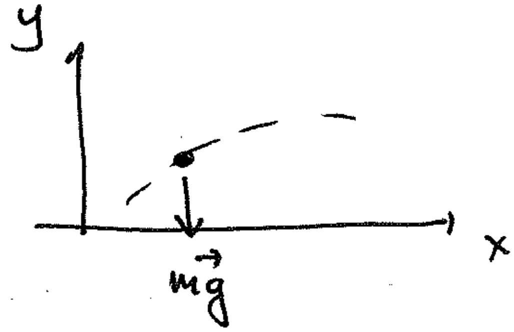
$$\cancel{y(2)}$$
$$x(2) = 32 \cdot 2 = 64 \text{ ft}$$

Max ht: (at time $t = 1$)

$$y(1) = 32 - 16 = 16 \text{ ft}$$

Projectile Motion

$$\vec{F} = \frac{d}{dt}(m\vec{v})$$
$$= m\vec{a}$$



$$\vec{r}(t) = x(t)\hat{i} + y(t)\hat{j}$$
$$\vec{r}'(t) = x'(t)\hat{i} + y'(t)\hat{j}$$
$$\vec{r}''(t) = x''(t)\hat{i} + y''(t)\hat{j}$$

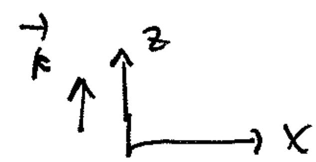
$$\vec{F} = m\vec{a}$$

$$-\cancel{m}g\hat{j} = \cancel{m}\vec{a}$$

$$g\hat{j} = x''(t)\hat{i} + y''(t)\hat{j}$$

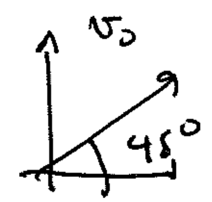
MKS units $g = 9.8\ m/sec^2$

Note



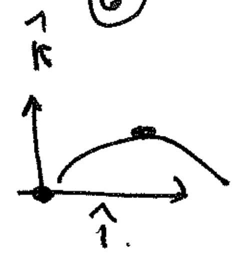
$$\vec{a}(t) = -9.8\hat{k}$$

$$\vec{v}(t) = \int_0^t \vec{a}(s)\,ds + \vec{v}(0)$$



$$= \int_0^t (-9.8\hat{k})\,ds +$$

$$v_0 \cos 45^\circ \hat{i} + v_0 \sin 45 \hat{k}$$



$$\vec{v}(t) = v_0 \frac{\sqrt{2}}{2}\,\hat\imath + v_0 \frac{\sqrt{2}}{2}\,\hat k$$

$$+ \ -9.8\,t\,\hat k$$

$$\vec{r}(t) = \vec{r}(0) + \int_0^t \vec{v}(s)\,ds$$

$$= \vec{0} + \int_0^t \left(v_0 \frac{\sqrt{2}}{2}\,\hat\imath + \left(v_0 \frac{\sqrt{2}}{2} - 9.8 s \right) \hat k \right) ds$$

$$\vec{v}(t) = v_0 t \frac{\sqrt{2}}{2}\,\hat\imath + \left(v_0 \frac{\sqrt{2}}{2} t - 9.8 \frac{t^2}{2} \right) \hat k$$

$t_0 =$ time hits the ground

$$\left.\begin{aligned} &(1) \quad 90 = v_0 t_0 \cdot \frac{\sqrt{2}}{2} \\ &(2) \quad v_0 \frac{\sqrt{2}}{2} - 9.8 \frac{t_0}{2} = 0 \end{aligned}\right\}$$

now solve for v_0

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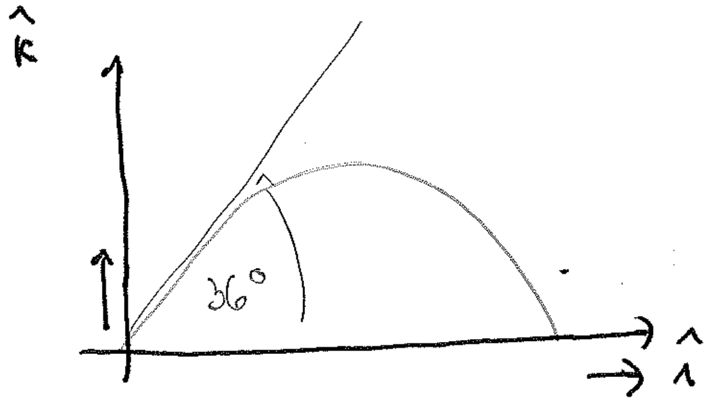


$$\vec{r}\,''(0) = -32\,\hat{k}$$

$$\vec{r}\,'(0) = v_0 \cos 36°\,\hat{\imath} + v_0 \sin 36°\,\hat{k}$$

$$\vec{r}\,'(t) = \vec{r}\,'(0) + \int_0^t \vec{r}\,''(s)\,ds$$

$$\vec{v}(t) \qquad \vec{v}(0) \qquad \int_0^t \vec{a}(s)\,ds$$

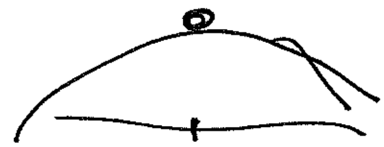
$$= v_0 \cos 36°\,\hat{\imath} + v_0 \sin 36°\,\hat{k} + \int_0^t -32\,\hat{k}\,ds$$

$$= v_0 \cos 36°\,\hat{\imath} + v_0 \sin 36°\,\hat{k} + -32t\,\hat{k}$$

$$\vec{r}(t) = \vec{r}(0) + \int_0^t \vec{r}\,'(s)\,ds$$

$$= \cancel{v_0 \cos 36°\,\hat{\imath} + v_0 \sin 36°\,\hat{k}}$$

$$\cancel{+\int_0^t -}$$



$$= \vec{0} + \int_0^t \left[\left(v_0 \cos 36^\circ \hat{\imath} \right) + \left(v_0 \sin 36^\circ - 32s \right) \hat{k} \right] ds$$

$$\vec{r}(t) = v_0 \cos(36^\circ) \cdot t \, \hat{\imath} + \left((v_0 \sin 36^\circ) \cdot t - 16t^2 \right) \hat{k}$$

① $$1600 = (v_0 \sin 36^\circ) t_{max} - 16 t_{max}^2$$

t_{max} = time of max height

② $$v_0 \sin(36^\circ) \cancel{t} - 32 t_{max} = 0$$