Functions of several Variables

$$f(x,y) = 3x + 2y + 4$$

$$z = 3x + 2y + 4$$

$$-3x - 2y + z = 4$$

$$g(x,y) = 2 - x - y$$

$$z = 2 - x - y \qquad \text{- graph}$$

$$x + y + z = 2$$

$$z = x^2 + y^2$$

Global min at

$$(x,y) = (0,0)$$

$$r^2 = x^2 + y^2$$

$$(x,y)$$

$$z = x^2 - y^2$$

xz plane: $z = x^2$
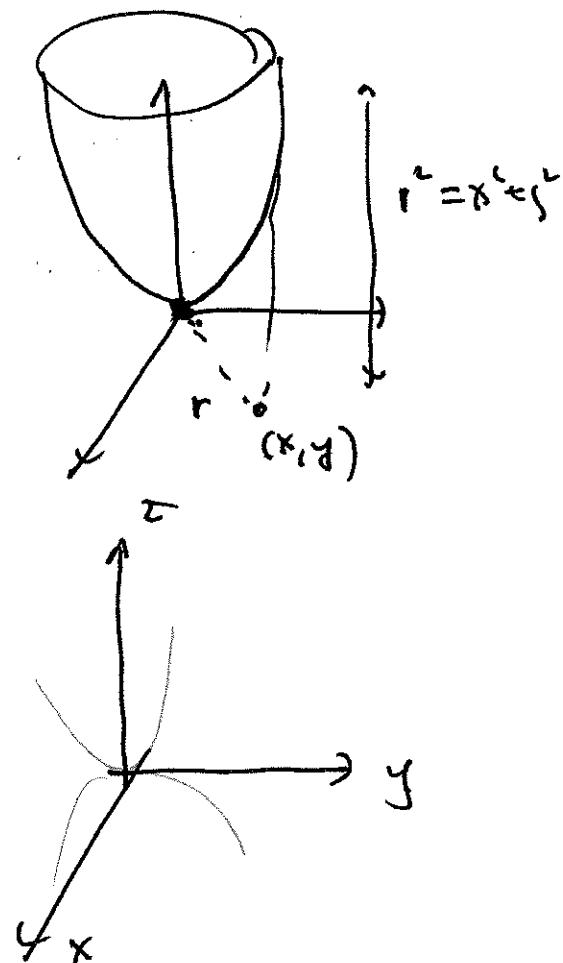
yz plane: $z = -y^2$

$$f(x,y) = \sqrt{4 - x^2 - y^2}$$

Domain: $\cancel{x^2 + y^2}$

$$4 - x^2 - y^2 \geq 0$$

$$4 \geq x^2 + y^2$$

$$x^2 + y^2 \leq 4$$

$$z = \sqrt{4 - x^2 - y^2}$$

$$z^2 = 4 - x^2 - y^2$$

$$x^2 + y^2 + z^2 = 4$$

graph:

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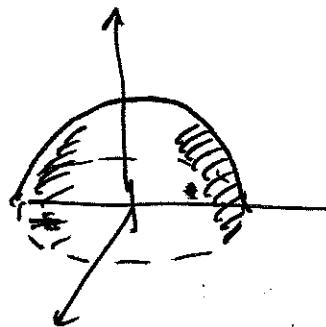
More graphs of functions.

$x^2 + y^2$ is the square of the distance of (x,y) from the origin.

$$f(x,y) = e^{-(x^2+y^2)} \qquad \text{Ran} f - (0,1]$$

$$f(x,y) = k$$

$$e^{-(x^2+y^2)} = k \qquad k \in (0,1]$$

$$\ln(\cdot) \qquad -(x^2+y^2) = \ln k$$

$$x^2+y^2 = -\ln k$$
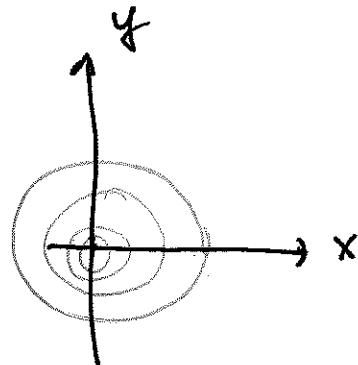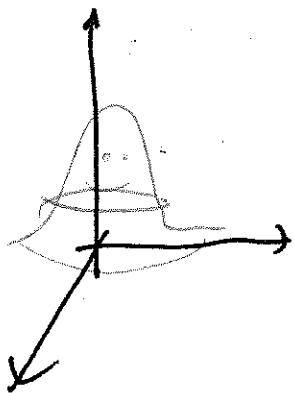
$$f(x,y) = x^2+y^2 \qquad \text{Ran } [0,\infty)$$

$$x^2+y^2 = k$$

Level Curves

Function	Level Curve

$f(x,y) = \sin(xy)$

$\sin(xy) = k$

$\arcsin k = xy$

$\rightarrow x \rightarrow -x \quad y \rightarrow -y$

no change

$f(x,y) = (1-x^2) \cdot (1-y^2)$

$f(x,y) = \sin(x-y)$

$\rightarrow \sin(x-y) = k$

$x-y = \arcsin k$

$x - y = c$

$y = c + x$

Three Variables

1: $f(x,y,z) = x^2 + y^2 + z^2$

Domain: $\mathbb{R}^3$

Range: $[0, \infty)$

2: $f(x,y,z) = \sqrt{9 - x^2 - y^2 - z^2}$

Domain: $9 - x^2 - y^2 - z^2 \geq 0$

$9 \geq x^2 + y^2 + z^2$

$x^2 + y^2 + z^2 \leq 9$

Range: $[0, 3]$

$$f(x,y,t) = x+y+z$$

$$D: \ \mathbb{R}^3$$

$$R: \ (-\infty, \infty)$$

$$f(x,y,t) = x^2 + y^2 + z^2$$

level surf: $x^2 + y^2 + z^2 = K$