$$f(x,y) = x^2 + xy^2$$

$\frac{\partial f}{\partial x}$ = rate of change of f wrt x holding y fixed

$\frac{\partial f}{\partial y}$ = " " " " f " y " x "

f_y $\frac{\partial f}{\partial y}$

f_x $\frac{\partial f}{\partial x}$

$$f(x,y) = x^2 + xy^2$$

$$\frac{\partial f}{\partial x}(x,y) = \frac{\partial}{\partial x}(x^2) + \frac{\partial}{\partial x}(xy^2)$$

$$= 2x + y^2$$

$$\frac{\partial f}{\partial y}(x,y) = \frac{\partial}{\partial y}(x^2) + \frac{\partial}{\partial y}(xy^2)$$

$$= 0 + x \cdot 2y$$

$$= 2xy$$

$$L(x,y) = f(a,b) + f_x(a,b)(x-a) + f_y(a,b)(y-b)$$

$$z = f(a,b) + f_x(a,b)(x-a) + f_y(a,b)(y-b)$$

$$-f_x(a,b)(x-a) - f_y(a,b)(y-b) + z = f(a,b)$$

$$f(x,y) = x^2 + y^2$$

$$f_x(x,y) = 2x$$

$$f_y(x,y) = 2y$$

$$a = 1 \quad b = 2$$

$$f_x(1,2) = 2 \qquad f(1,2) = 1^2 + 2^2 = 5$$

$$f_y(1,2) = 4$$

$$L(x,y) = 5 + 2(x-1) + 4(y-2)$$

$$z = 5 + 2(x-1) + 4(y-2)$$

$$-2(x-1) - 4(y-2) + z = 5$$

① $f(x,y) = x^{y^{y^{y^{y}}}} \sin(x)$

$$f_x(x,1) = \frac{d}{dx}\left[f(x,1)\right]$$

$$= \frac{d}{dx}\left[x \sin x\right]$$

$$= \sin x + x \cos x$$

② $f_y(\cancel{y}\,3, y) = \frac{d}{dy}\left[f(3,y)\right]$

$$= \frac{d}{dy}\left[3y\right]$$

$$= 3$$

$$f(x,y) = x^2 y^2$$

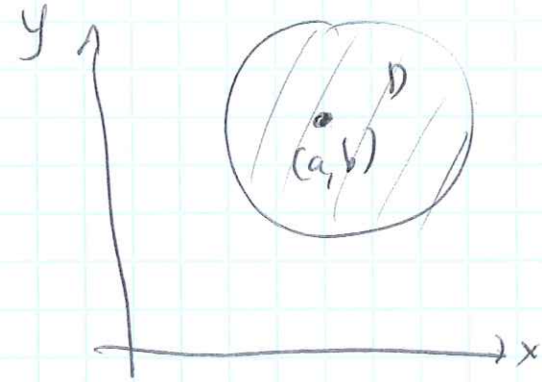
$$\frac{\partial f}{\partial x}(x,y) = 2xy^2 \qquad \frac{\partial f}{\partial y} = 2x^2 y$$

$$\frac{\partial^2 f}{\partial x^2} = \frac{\partial}{\partial x}\left(\frac{\partial f}{\partial x}\right) = \frac{\partial}{\partial x}\left(2xy^2\right) = 2y^2$$

$$\boxed{\frac{\partial}{\partial y}\left(\frac{\partial f}{\partial x}\right) = \frac{\partial}{\partial y}\left(2xy^2\right) = 4xy}$$

$\downarrow$

$$\boxed{\frac{\partial}{\partial x}\left(\frac{\partial f}{\partial y}\right) = \frac{\partial}{\partial x}\left(2x^2 y\right) = 4xy}$$



$$f_{xy} = \frac{\partial^2 f}{\partial y\, \partial x} \qquad f_{yx} = \frac{\partial^2 f}{\partial x\, \partial y}$$

$$= \frac{\partial}{\partial y}\left(\frac{\partial f}{\partial x}\right) \qquad = \frac{\partial}{\partial x}\left(\frac{\partial f}{\partial y}\right)$$

$$f(x,y) = x^3 y^2 - \sin(xy)$$

$$\frac{\partial f}{\partial x} = \underline{3x^2y^2 - y\cos(xy)}$$

$$\frac{\partial f}{\partial y} = \underline{\underline{2x^3 y - x\cos(xy)}}$$

$$f_{xy} = \frac{\partial}{\partial y}\left(\frac{\partial f}{\partial x}\right) = \underline{6x^2y - \cos(xy) - y(-\sin(xy)\cdot x)}$$

$$f_{yx} \quad \frac{\partial}{\partial x}\left(\frac{\partial f}{\partial y}\right) = \underline{\underline{6x^2y - \cos(xy) - x(-\sin(xy)\cdot y)}}$$

Implicit Differentiation

z is defined implicitly as a function of x, y

$$x^2 + y^2 + z^2 = 1 \qquad -(1)$$

$$\frac{\partial z}{\partial x}: \quad 2x + 0 + 2z \boxed{\frac{\partial z}{\partial x}} = 0$$

$$2z \frac{\partial z}{\partial x} = -2x$$

$$\frac{\partial z}{\partial x} = -\frac{2x}{2z} = -\frac{x}{z}$$

$$\frac{\partial z}{\partial y}: \quad 0 + 2y + 2z \frac{\partial z}{\partial y} = 0$$

$$\frac{\partial z}{\partial y} = -\frac{2y}{2z} = -\frac{y}{z}$$

(2) $e^z = xyz$

Find $\frac{\partial z}{\partial x}$, $\frac{\partial z}{\partial y}$

$$\frac{\partial z}{\partial x}(\cdot): \quad e^z \boxed{\frac{\partial z}{\partial x}} = yz + xy \boxed{\frac{\partial z}{\partial x}}$$

$$(e^z - xy) \boxed{\frac{\partial z}{\partial x}} = yz$$

$$\frac{\partial z}{\partial x} = \frac{yz}{e^z - xy}$$