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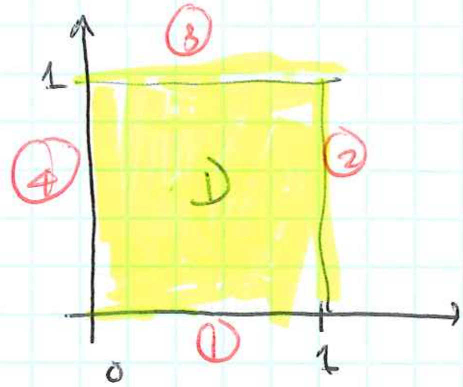
Closed Set Method - Example

$$f(x,y) = x^2 + 4xy + y^2 + 6x$$

on $D = [0,1] \times [0,1]$



1 Critical Point

(1) $f_x(x,y) = 2x + 4y + 6$

(2) $f_y(x,y) = 2y + 4x$

From (2) $\quad 2y + 4x = 0 \quad$ or $\quad \boxed{y = -2x}$

use in (1) $\quad 2x + 4(-2x) + 6 = 0$

$$-6x + 6 = 0$$

$$x = 1$$

$$y = -2$$

No CP. in D

2 on 1 $\quad \cancel{g_1(x,0) =} \; g_1(x) = f(x,0) = x^2 + 6x$

$$0 \le x \le 1$$

$g_1'(x) = 2x + 6 \qquad$ No CP in $[0,1]$

$g_1(0) = 0$

$g_1(1) = 7$

$g_2(y) = f(1,y) = y^2 + 4y + 7$

$g_3(x) = f(x,1) = x^2 + 10x + 1$

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$\boxed{g_4(y) = y^2}$

(2) $g_2(y) = y^2 + 4y + 7$

$g_2'(y) = 2y + 4 \Rightarrow y = -2$ no CP in [0,1]

$g_2(0) = 7$

$g_2(1) = 12$

(3) $g_3(x) = x^2 + 10x + 1$

$g_3'(x) = 2x + 10 \Rightarrow x = -5$ no CP in [0,1]

$g_3(0) = 1$

$g_3(1) = 12$

(4) $g_4(y) = y^2$ $g_4'(y) = 2y$ $y = 0$ is a CP

$g_4(0) = 0$

$g_4(1) = 1$

$\therefore$ Min is 0, Max is 12

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Lagrange Multipliers

Constrained Max / Min Problem

Ex: Find the max/min of $\boxed{f(x,y) = x^2 - y^2}$

on the circle $\boxed{x^2+y^2=1}$

$\uparrow$

constraint

Minimize / Maximize f along a curve $(x(t), y(t))$, you find min/max of

$$\phi(t) = f(x(t), y(t))$$

so look for the ~~pa~~ values of t that make

$$\phi'(t) = 0$$

$$\frac{d\phi}{dt} = \frac{\partial f}{\partial x} \cdot \frac{dx}{dt} + \frac{\partial f}{\partial y} \frac{dy}{dt}$$

$$= \left(\nabla f(x(t), y(t))\right) \cdot \left(x'(t), y'(t)\right)$$

$\uparrow$ gradient of $\qquad$ $\uparrow$ tangent to level curve

$= 0$

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So max/min occurs when ∇f is $\perp$ to the level.

But ∇g is also $\perp$ to the level curve

so $\nabla f = \lambda \nabla g$

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$$f(x,y) = x^2 - y^2$$

$$g(x,y) = x^2 + y^2 - 1 = 0$$

$$\nabla f = \langle 2x, -2y \rangle$$

$$\nabla g = \langle 2x, 2y \rangle$$

$$\nabla f = \lambda \nabla g$$

$$\left.\begin{array}{ll} (1) & 2x = \lambda \cdot 2x \\ (2) & -2y = 2\lambda y \end{array}\right\} \quad \nabla f = \lambda \nabla g$$

(3) $x^2 + y^2 = 1$

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(1)' $2x(\lambda - 1) = 0$ $\Rightarrow$ either $x = 0$ or $\lambda = 1$

(2)' $2y(\lambda + 1) = 0$ $\rightarrow$ either $y = 0$ or $\lambda = -1$

(3)' $x^2 + y^2 = 1$

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If $x=0$, ~~then~~ and $\lambda = -1$

then $(1)'$, $(2)'$ satisfied.

By (3), $0^2+y^2=1$ or $y=\pm 1$

$\therefore$ $(0,1)$, and $(0,-1)$ are pts. to be tested

If $y=0$ and $\lambda=1$ then

$x=\pm 1$

$(+1,0)$, $(-1,0)$ are pts to be tested

x	y	$f(x,y) = x^2-y^2$
0	1	-1
0	-1	-1
1	0	1
-1	0	1

#2 $f(x,y) = 3x+y$

$g(x,y) = x^2+y^2-10$

Lagrange eq's $\nabla f = \lambda \nabla g$ 2 eq'ns

$g(x,y)=0$ 1 eq'n

$\nabla f(x,y) = \langle 3, 1\rangle$

$\nabla g(x,y) = \langle 2x, 2y\rangle$

$\nabla f = \lambda \nabla g$:

(1) $3 = 2\lambda x$

(2) $1 = 2\lambda y$

$g(x,y)=0$

(3) $x^2+y^2-10=0$

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(1)' $\lambda = \frac{3}{2x}$ (2)' $\lambda = \frac{1}{2y}$

$\therefore \frac{3}{2x} = \frac{1}{2y} \Rightarrow \frac{2x}{3} = 2y \Rightarrow y = \frac{1}{3}x$

(3)' $x^2 + \left(\frac{1}{3}x\right)^2 - 10 = 0$

10/9/2019 (5)

(1), (4) $\Rightarrow$ $y = \frac{1}{3}x$

(3)' $x^2 + (\frac{1}{3}x)^2 = 10$

$$x^2 + \frac{x^2}{9} = 10$$

$$\frac{10}{9}x^2 = 10$$

$$x^2 = 9$$

$$x = \pm 3$$

$$y = \pm 1$$

x	y	$f(x,y) = 3x+y$	
3	1	$\boxed{10}$	max
-3	1	-8	
3	-1	8	
-3	-1	$\boxed{-10}$	min

$f(x,y,z) = e^{xyz}$

$g(x,y,t) = 2x^2+y^2+z^2=24$

$\nabla f = \langle yze^{xyz},\ xze^{xyz},\ xye^{xyz} \rangle$

$\nabla g = \langle 4x, 2y, 2z \rangle$

(1) $yze^{xyz} = 4\lambda x$ $\qquad \lambda = \frac{yz}{4x} e^{xyz}$

(2) $xze^{xyz} = 2\lambda y$ $\qquad \lambda = \frac{xz}{2y} e^{xyz}$

(3) $xye^{xyz} = 2\lambda z$ $\qquad \lambda = \frac{xye^{xyz}}{2z}$

(4) $2x^2+y^2+z^2=24$

From (1), (2), (3) $\quad \frac{yz}{4x} \cancel{e^{xyz}} = \frac{xz}{2y} \cancel{e^{xyz}} = \frac{xy}{2z} \cancel{e^{xyz}}$

so $\quad \frac{y\cancel{z}}{4x} = \frac{x\cancel{z}}{2y} \qquad \frac{\cancel{x}z}{2y} = \frac{\cancel{x}y}{2z}$

$4x^2 = 2y^2 \qquad 2z^2 = 2y^2$

$$\begin{cases} 4x^2 = 2y^2 \\ 2z^2 = 2y^2 \\ 2x^2 + y^2 + z^2 = 24 \end{cases}$$

$x^2 = \frac{1}{2}y^2$

$z^2 = y^2$

$y^2 + y^2 + y^2 = 24$

$3y^2 = 24$

$y^2 = 8$

$\boxed{y = \pm 2\sqrt{2}}$

$x^2 = \frac{1}{2}(8) = 4$ $\quad x = \pm 2$

$z^2 = 8$ $\quad z = \pm 2\sqrt{2}$

$f(x, y, z)$

x	y	z
2	$2\sqrt{2}$	$2\sqrt{2}$

#10 due tonight BJ

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