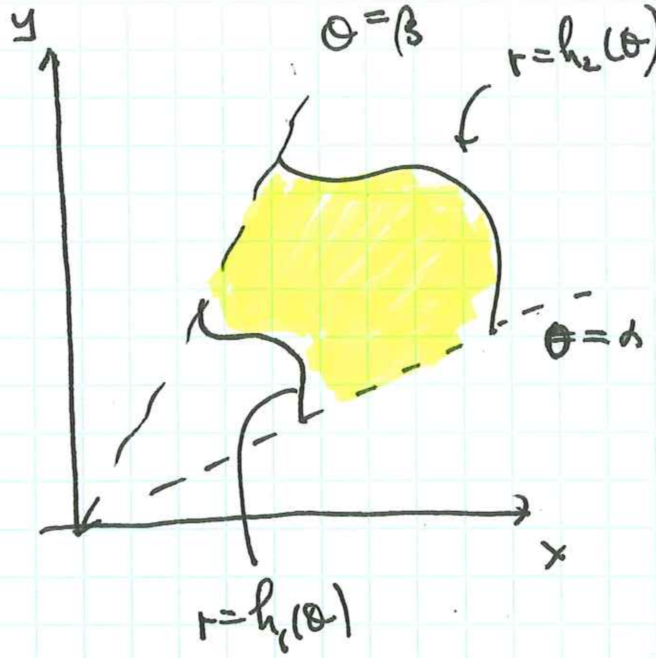
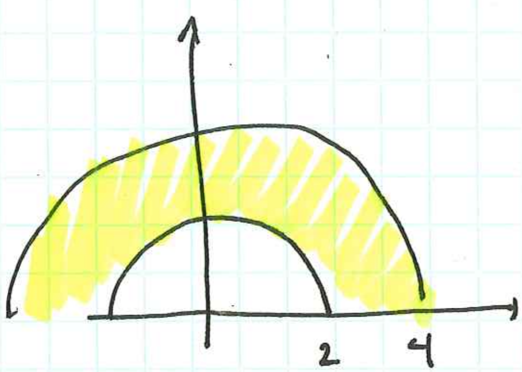


$D = \{(r,\theta) : \alpha \le \theta \le \beta,$
$h_1(\theta) \le r \le h_2(\theta)\}$

Ex:



$D = \{(r,\theta) : 0 \le \theta \le \pi,$
$2 \le r \le 4\}$

$$\overset{(3)}{\int_0^1} \left(\overset{(2)}{\int_0^1} \left(\overset{(1)}{\int_0^{2-x^2-y^2}} xy\, e^z \, dz \right) dy \right) dx$$

$$E = \{(x, y, z) :$$

$$0 \le x \le 1,$$

$$0 \le y \le 1,$$

$$0 \le z \le 2 - x^2 - y^2\}$$

(1) $$\int_0^{2-x^2-y^2} xy\, e^z \, dz =$$

$$xy \left[e^z \right]_{z=0}^{z=2-x^2-y^2} =$$

$$xy \left[e^{(2-x^2-y^2)} - 1 \right]$$

$$e^{2-x^2-y^2} = e^{2-x^2} \cdot e^{-y^2}$$

② $\int_0^1 xy\left[e^{2-x^2-y^2} - 1\right] dy =$

$$xe^{2-x^2} \int_0^1 ye^{-y^2}\, dy - \int_0^1 xy\, dy =$$

$$xe^{2-x^2}\left[\int_0^1 \tfrac{1}{2} e^{-u}\, du\right] - \left[\frac{xy^2}{2}\Big|_0^1\right] =$$

$u = y^2 \quad du = 2y\,dy$

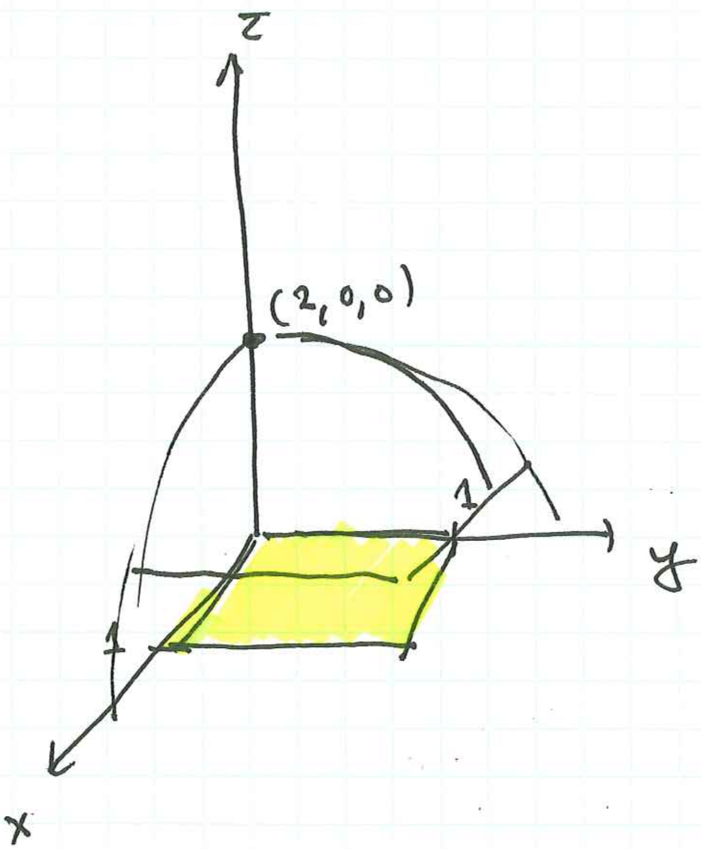
$y = 0 \Rightarrow u = 0$

$y = 1 \Rightarrow u = 1$

$$xe^{2-x^2}\left(-\tfrac{1}{2}e^{-u}\Big|_0^1\right) - \frac{x}{2} =$$

$$xe^{2-x^2}\left(\tfrac{1}{2}(1-e^{-1})\right) - \frac{x}{2}$$

③ $\int_0^1 \left(\left[xe^{2-x^2}\left(\tfrac{1}{2}(1-e^{-1})\right)\right) - \frac{x}{2}\right) dx$



$0 \le z \le 2 - x^2 - y^2$

$$\begin{cases} z = 2 - x^2 - y^2 \\ z = 0 \end{cases}$$

graph: $z = 2 - x^2 - y^2$

Trace in xz plane $y = 0$

$z = 2 - x^2$

Trace in yz plane $x = 0$

$z = 2 - y^2$

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$$D = \{ 0 \le y \le 1, \quad y \le x \le 1 \}$$

$$0 \le z \le xy$$

$$\iiint_E y \, dV = \iint_D \left(\int_0^{xy} y \, dz \right) dA$$

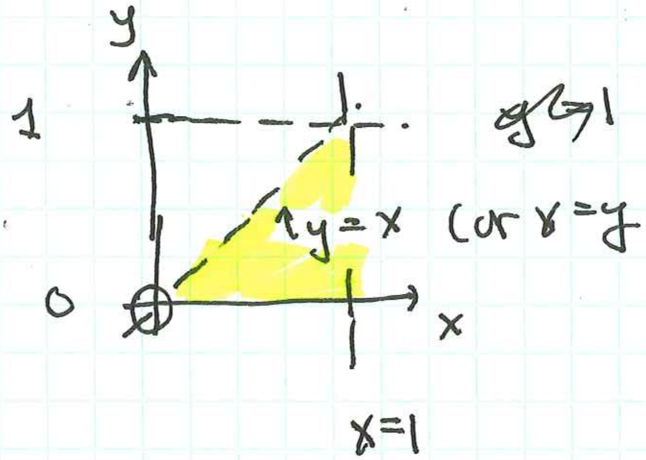
$$= \int_0^1 \left(\int_y^1 \left(\int_0^{xy} y \, dz \right) dx \right) dy$$

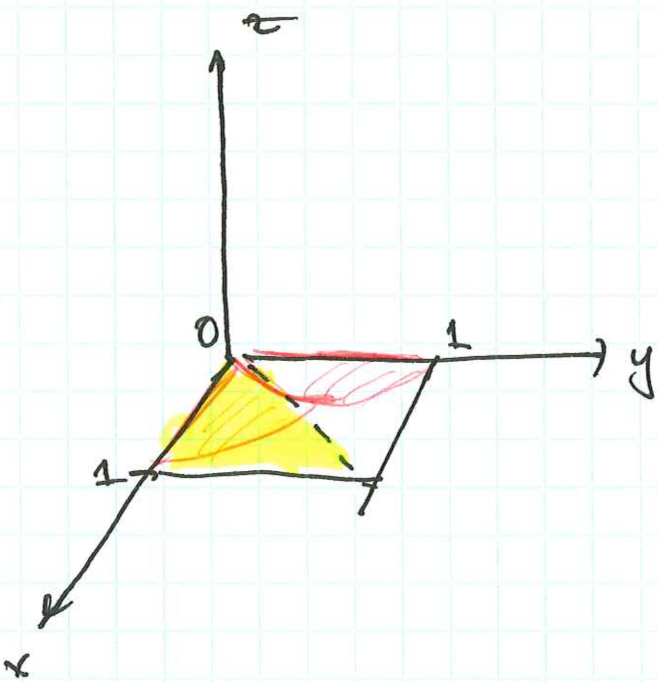
10/25/2019 (5)

Type I

$$E = \{ (x, y) \in D : \quad 0 \le z \le xy \}$$

$$= \{ 0 \le y \le 1, \quad y \le x \le 1, \quad 0 \le z \le xy \}$$





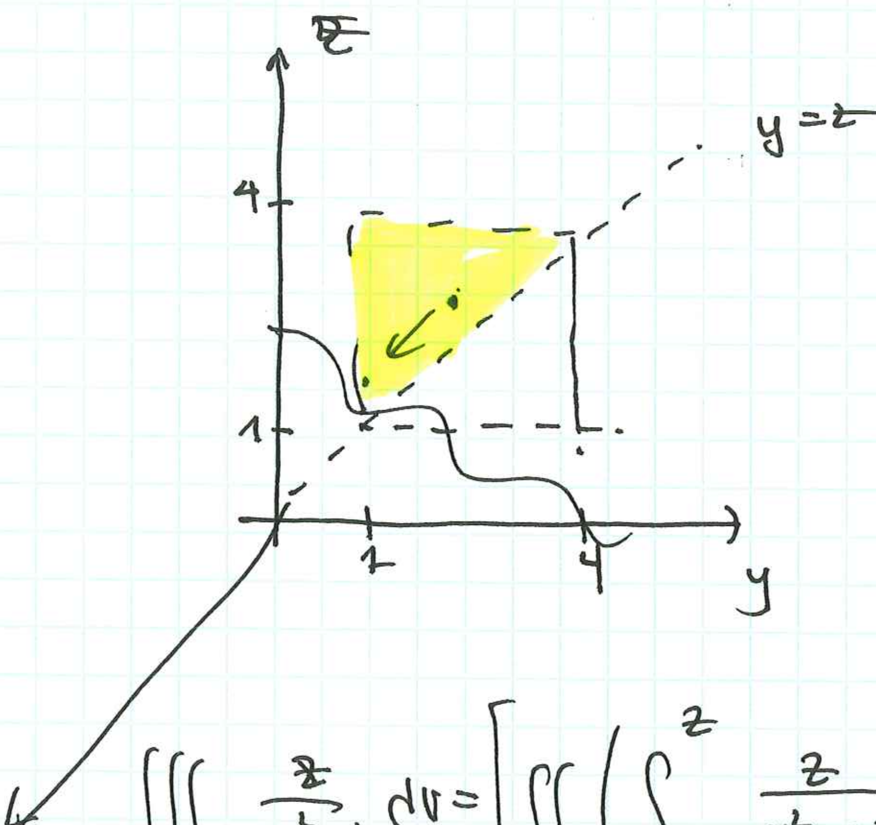
$z = xy$

Type II   10/25/2019 7

$$E = \{(x,y,z) : 1 \le y \le 4,\ y \le z \le 4,\ \underline{0 \le x \le z}\}$$

$$D = \{(y,z) : 1 \le y \le 4,\ y \le z \le 4\}$$

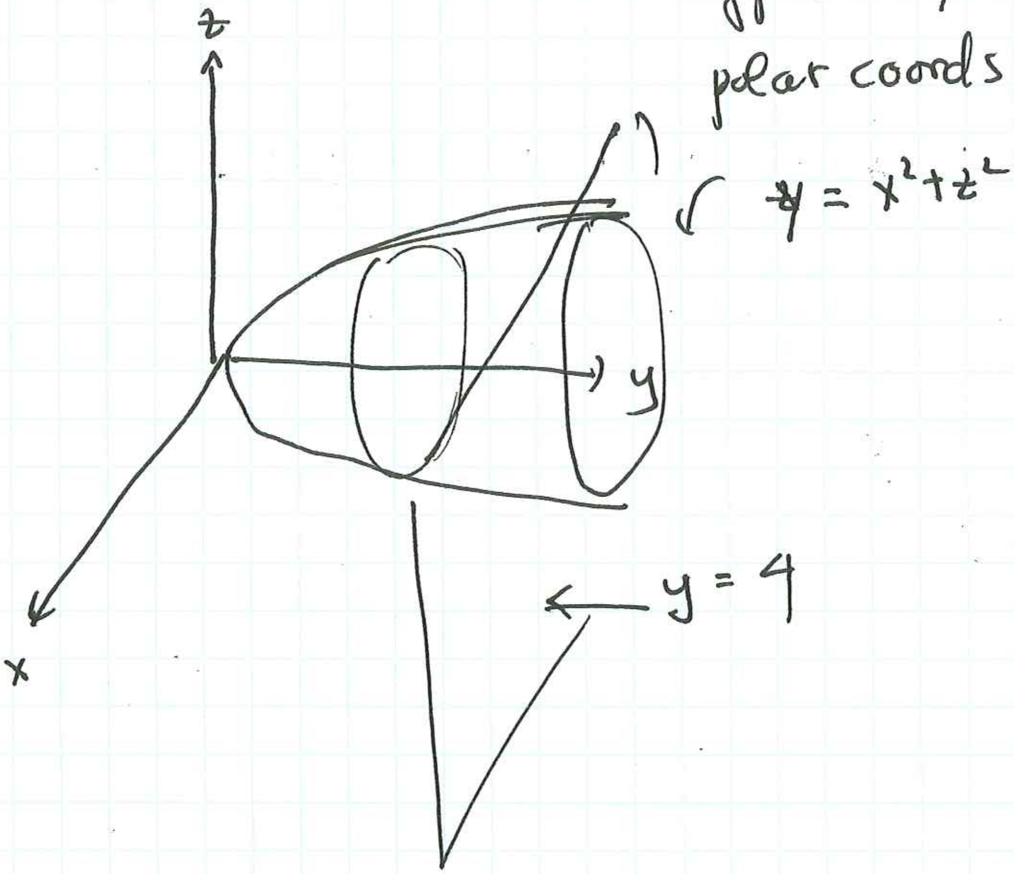
$$\iiint_E \frac{z}{x^2+z^2}\, dV = \iint_D \left(\int_0^z \frac{z}{x^2+z^2}\, dx \right) dA$$



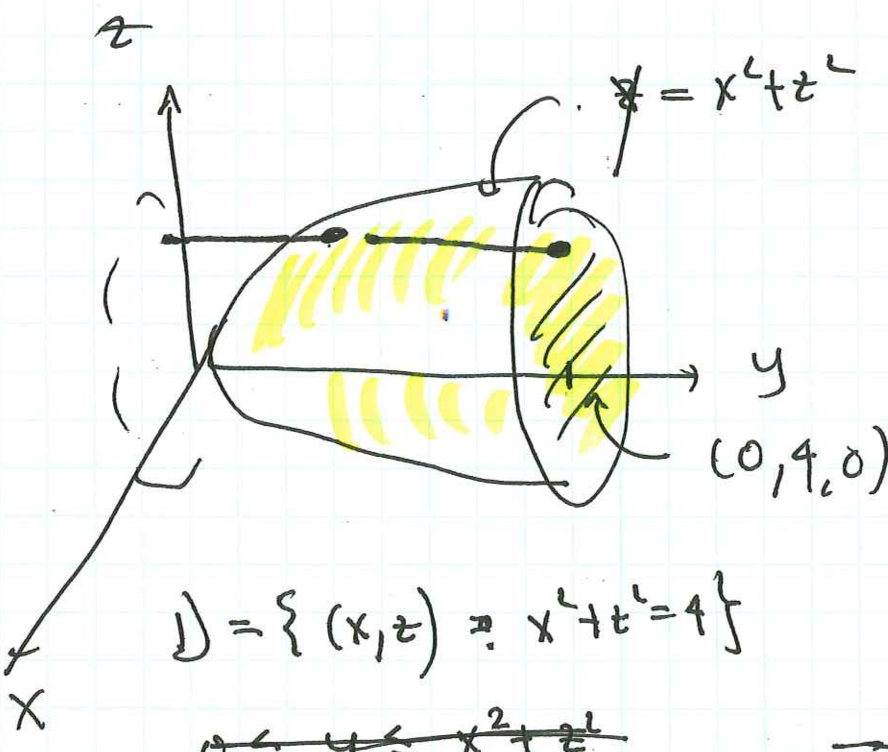
$$\iiint_E \frac{z}{x^2+z^2}\, dV = \left[\iint_D \left(\int_0^z \frac{z}{x^2+z^2}\, dx \right) dA \right]$$

$$= \int_1^4 \left(\int_y^4 \left(\int_0^z \frac{z}{x^2+z^2}\, dx \right) dz \right) dy$$

Type III w/
polar coords



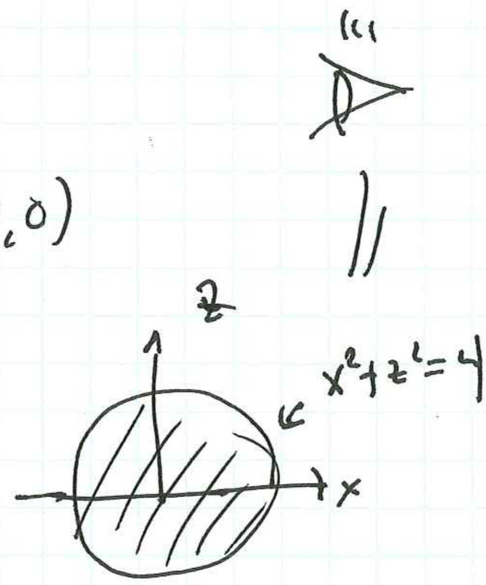
$\Downarrow$



$D = \{ (x,z) : x^2+z^2 = 4 \}$

~~$0 \le y \le x^2+z^2$~~

~~$x^2 \le y$~~ $x^2+z^2 \le y \le 4$



$$D = \{(x,z) : x^2 + z^2 \le 4\}$$

$$E = \{(x,y,z) : (x,z) \in D,$$
$$x^2 + z^2 \le y \le 4\}$$

let $x = r\cos\theta$

$z = r\sin\theta$

$$x^2 + z^2 = r^2\cos^2\theta + r^2\sin^2\theta = r^2$$

$$\iint_D \left(\int_{x^2+z^2}^{4} \sqrt{x^2+z^2}\, dy \right) dA =$$

$$\iint_D \left(\int_{r^2}^{4} r\, dy \right) dA =$$

$$\iint_D r(4 - r^2)\, dA =$$

$$\int_0^{2\pi} \left(\int_0^2 r(4-r^2)\, r\, dr \right) d\theta$$

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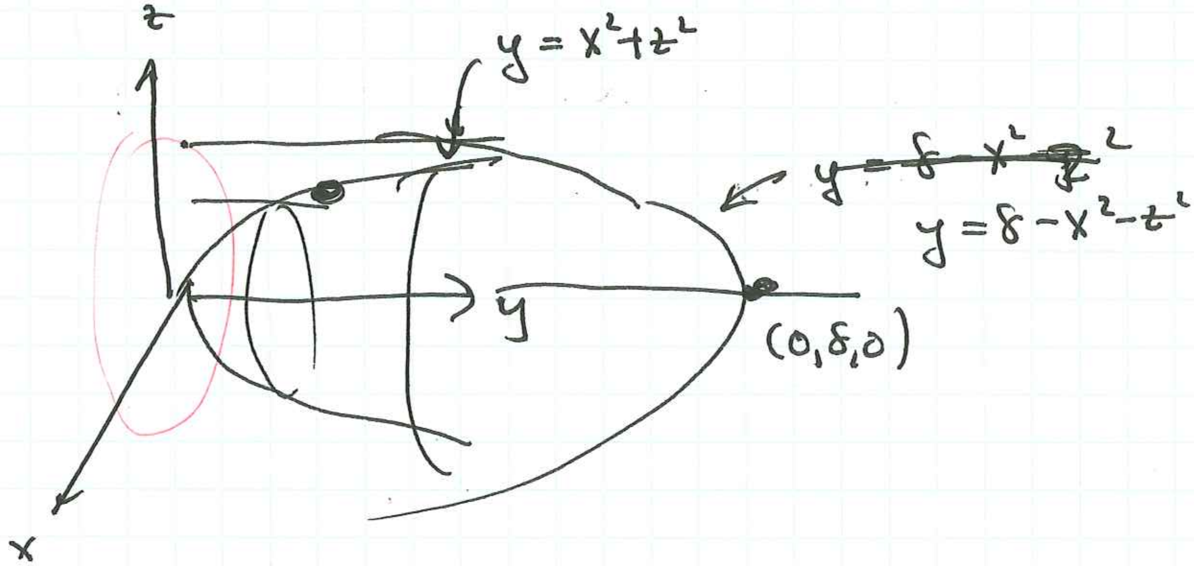

$$x^2 + z^2 \le y \le 8 - x^2 - z^2$$

$$x^2 + z^2 \le 4$$

$x = r\cos\theta$

$z = r\sin\theta$

$\cancel{x^2 + y}$ $x^2 + z^2 = r^2$

$$\iint_D \left(\int_{x^2+z^2}^{8-x^2-z^2} dy \right) dA =$$

$$\iint_D \left(\int_{r^2}^{8-r^2} dy \right) dA =$$

$$\int_0^{2\pi}\int_0^{2}\left(8-2r^2\right)r\,dr\,d\theta$$