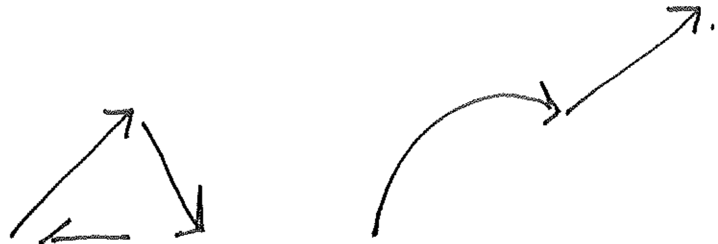
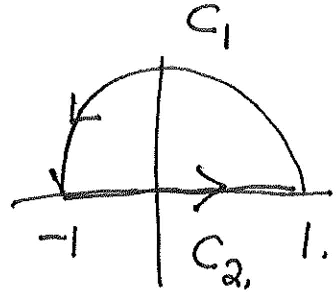
Line integrals II, cont'd.

curve
C = union of piecewise smooth functions $= \bigcup_i C_i$

ex.



ex. $\displaystyle\int_C xy\, ds$ for



C_1: $x = \cos t$, $y = \sin t$, $\dfrac{dx}{dt} = -\sin t$, $\dfrac{dy}{dt} = \cos t$

$$= \sum_i \int_{C_i} xy\, ds$$

$$= \int_{-\pi/2}^{\pi/2} (\cos t)(\sin t)\cdot \sqrt{s^2+c^2}\; dt$$

$$+ \int_{-1}^{1} t\cdot 0\,\sqrt{1^2+0^2}\; dt = \int_{-1}^{1} 0\, dt = 0.$$

C_2: $x = t$, $-1 \le t \le 1$
$y = 0$

Line integral of f
wrt x (wrt y)

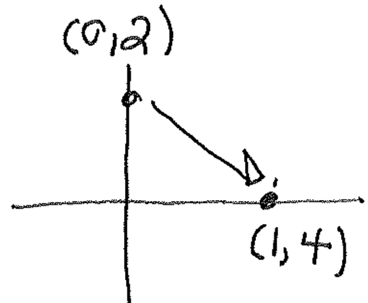
$$\int_C f(x,y)\,dx = \int_a^b f(x(t),y(t))\;x'(t)\;dt$$

$$\int_C f(x,y)\,dy = \int_a^b f(x(t),y(t))\;y'(t)\;dt$$

ex. $\cancel{\int_C x^2\,dx}$

shorthand

$$\int_C P\,dx + Q\,dy = \int_C P\,dx + \int_C Q\,dy$$



ex. $\displaystyle\int_C \sin(\pi y)\,dy + yx^2\,dx.$

$\vec{r}(t) = (1-t)\langle 0,2\rangle + t\langle 1,4\rangle.$
$0 \le t \le 1$
$= \langle t,\ 2+2t\rangle$

$$= \int_0^1 \sin(\pi(t))\cdot 2\,dt + \int_0^1 (2+2t)t^2(1)\,dt$$

$= \ldots = 7/6.$

ex.

$$\int_{-C} \text{same} = \text{''} = -\frac{7}{6}$$

$\vec{r}(t) = \langle 1-t, 4-2t \rangle$

↑ switch direction.

$$\left.\begin{aligned} \int_{-C} f\,dx &= -\int_{C} f\,dx \\ \int_{-C} f\,dy &= -\int_{C} f\,dy \end{aligned}\right\} \quad \int_{-C} P\,dx + Q\,dy$$

$$\|$$

$$-\int P\,dx + Q\,dy$$