Math 213 - Exam III Review

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Reminders

- Exam III is tonight, 5:00-7:00 PM
- Homework C7 is due on Friday Night
- Thanksgiving is coming!

Unit III: Multiple Integrals, Vector Fields

Double Integrals in Polar Coordinates
Triple Integrals (Part I)
Triple Integrals (Part II)
Triple Integrals in Cylindrical Coordinates
Triple Integrals in Spherical Coordinates
Change of Variables, Part I
Change of Variables, Part II

Vector Fields
Line Integrals (Scalar functions)
Line Integrals (Vector functions)

Exam III Review

Goals of the Day

- Learn how to get your best score yet on a Calculus Exam

Fantastic Integrals and How to Compute Them

Double Integral $\iint_D f(x,y)\,dA$	Iterated integral (Cartesian coordinates) Type I Region Type II Region Iterated integral (polar coordinates) Change of variables theorem
Triple Integral $\iiint_E f(x,y,z)\,dV$	Iterated integral in xyz over xy, xz, or yz plane Iterated integral (cylindrical coordinates)) Iterated integral (spherical coordinates) Change of variables theorem
Line integral $\int_C f(x,y)\,ds$	$ds = \sqrt{(x')^2 + (y')^2}\,dt$
Line integral $\int_C f(x,y)\,dx$	$dx = x'(t)dt$
Line integral $\int_C f(x,y)\,dy$	$dy = y'(t)dt$
Line integral $\int_C f(x,y,z)\,ds$	$ds = \sqrt{(x'(t))^2 + (y'(t))^2 + (z'(t))^2}\,dt$

Transformations: More Than Meets the Eye

In two dimensions, a *transformation* is a map

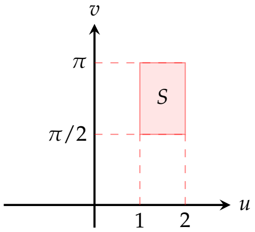
$$(u,v) \to (x(u,v), y(u,v))$$

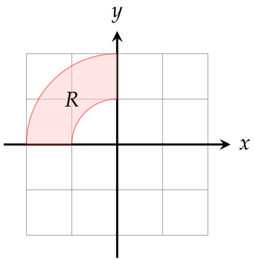
that transforms a region S of the uv plane to a region R of the xy plane. The 'volume change' factor is the absolute value of the *Jacobian*

$$\frac{\partial(x,y)}{\partial(u,v)} = \begin{vmatrix} \dfrac{\partial x}{\partial u} & \dfrac{\partial x}{\partial v} \\ \dfrac{\partial y}{\partial u} & \dfrac{\partial y}{\partial v} \end{vmatrix}$$

Example:

$$\begin{aligned} x(u,v) &= u\cos v \\ y(u,v) &= u\sin v \end{aligned}$$





Transformations: More Than Meets the Eye

In three dimensions, a *transformation* is a map

$$(u, v, w) \to (x(u, v, w), y(u, v, w), z(u, v, w))$$

that transforms a region S of uvw space to a region R of xyz space. The 'volume change factor' is the absolute value of the *Jacobian*

$$\frac{\partial(x, y, z)}{\partial(u, v, w)} = \begin{vmatrix} \dfrac{\partial x}{\partial u} & \dfrac{\partial x}{\partial v} & \dfrac{\partial x}{\partial w} \\ \dfrac{\partial y}{\partial u} & \dfrac{\partial y}{\partial v} & \dfrac{\partial y}{\partial w} \\ \dfrac{\partial z}{\partial u} & \dfrac{\partial z}{\partial v} & \dfrac{\partial z}{\partial w} \end{vmatrix}$$

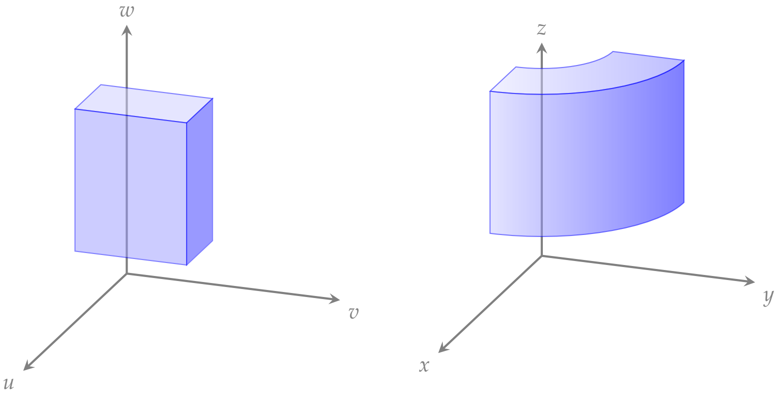
Example:

$$(u, v, w) \to (u \cos v, u \sin v, w)$$

Transformations: More Than Meets the Eye

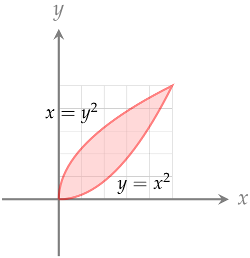
Example:

$$(u, v, w) \to (u\cos v, u\sin v, w)$$

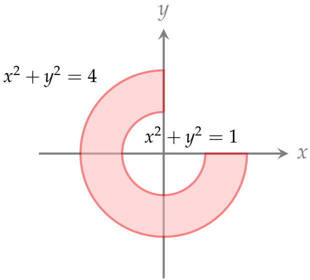


Double Integrals

Find the $\iint_D xy^2\,dA$ if D is the region bounded by the curves $y = x^2$ and $x = y^2$ in the first quadrant.



Find the $\iint_D xy\,dA$ if D is the region shown.



Triple Integrals, Cartesian Coordinates

Set up $\iiint_E f(x,y,z)\,dV$ six different ways if

$$E = \{(x,y,z) : 0 \le y \le 1,\ y \le x \le 1,\ 0 \le z \le y\}$$

$$\int_0^1 \int_y^1 \int_0^y f(x,y,z)\,dz\,dx\,dy = \int_0^1 \int_0^x \int_0^y f(x,y,z)\,dz\,dy\,dx$$

$$\int_0^1 \int_0^x \int_0^y f(x,y,z)\,dz\,dy\,dx = \int_0^1 \int_0^x \int_z^x f(x,y,z)\,dy\,dz\,dx$$

$$\int_0^1 \int_0^x \int_0^y f(x,y,z)\,dz\,dy\,dx = \int_0^1 \int_y^1 \int_z^x f(x,y,z)\,dy\,dx\,dz$$

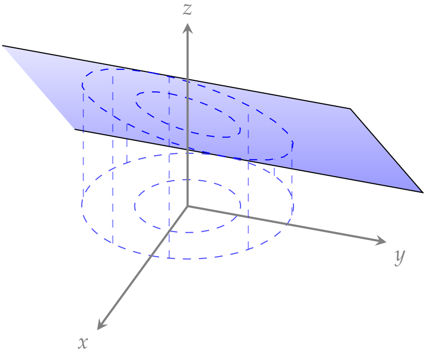
etc.

Triple Integrals, Cylindrical Coordinates

For cylindrical coordinates

$$dV = r\, dr\, d\theta\, dz$$

Find $\iiint_E y\, dV$ if E lies below the plane $z = x + 2$, above the xy plane, and between the cylinders $x^2 + y^2 = 1$ and $x^2 + y^2 = 4$.

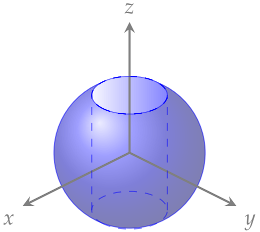


Triple Integrals, Spherical Coordinates

For spherical coordinates,

$$dV = \rho^2 \sin\phi \, d\rho \, d\theta \, d\phi$$

Find the volume of the region contained inside the sphere $x^2 + y^2 + z^2 = 4$ but outside the cylinder $x^2 + y^2 = 1$



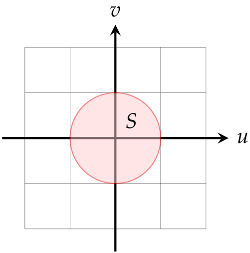
Change of Variables Theorem

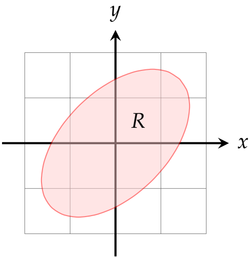
$$\iint_R f(x,y)\,dA = \iint_S f(x(u,v),y(u,v)) \left|\frac{\partial(x,y)}{\partial(u,v)}\right| du\,dv$$

Using the change of variables

$$x = \sqrt{2}u - \sqrt{\frac{2}{3}}v, \quad y = \sqrt{2}u + \sqrt{\frac{2}{3}}v$$

find $\iint_D (x^2 - xy + y^2)\,dA$ if D is the region $x^2 - xy + y^2 \leq 2$.





Line Integral Formulas

Arc length differential: $ds = \sqrt{x'(t)^2 + y'(t)^2 + z'(t)^2}\, dt$

Line integral with respect to arc length

$$\int_C f(x,y,z)\, ds = \int_a^b f(x(t),y(t),z(t))\, \sqrt{x'(t)^2 + y'(t)^2 + z'(t)^2}\, dt$$

Line integral with respect to x, y, z

$$\int_C f(x,y,z)\, dx = \int_a^b f(x(t),y(t),z(t)) x'(t)\, dt$$
$$\int_C f(x,y,z)\, dy = \int_a^b f(x(t),y(t),z(t)) y'(t)\, dt$$
$$\int_C f(x,y,z)\, dy = \int_a^b f(x(t),y(t),z(t)) z'(t)\, dt$$

Line integral of a vector field

$$\int_C \mathbf{F} \cdot d\mathbf{r} = \int_a^b \mathbf{F}(x(t),y(t),z(t)) \cdot \mathbf{r}'(t)\, dt$$

Line Integral Problems

Find $\int_C 4x^3\,ds$ if C is a line segment from $(-2,1)$ to $(1,2)$

Find $\int_C xy\,ds$ if C is a circle of radius 2.

Find $\int_C xy\,ds$ if C is the helix $\langle \cos(t), \sin(t), 2t \rangle$ from $t = 0$ to $t = \pi$.

Find $\int_C y\,dx$ if C is the quarter-circle of radius 3 in the first quadrant with center at the origin.