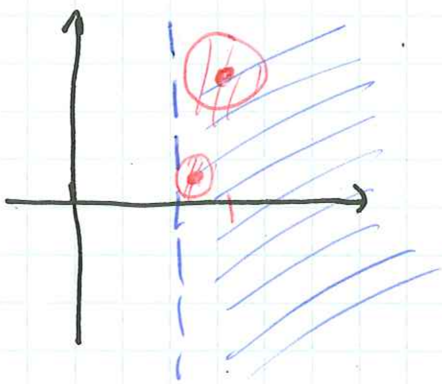
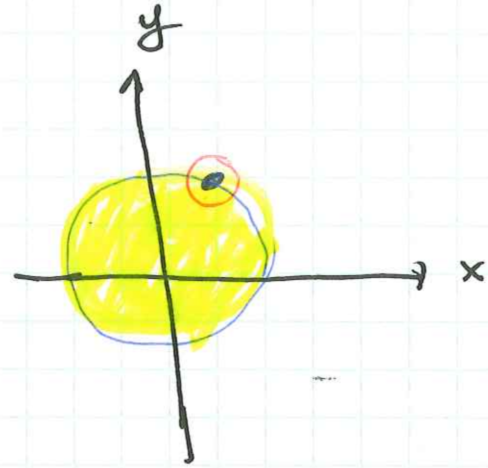
$$\int_C \vec{F} \cdot d\vec{r}$$



open regia



not open regia

$$\frac{d}{dt} f(x(t), y(t), z(t)) =$$

$$\frac{\partial f}{\partial x} \cdot \frac{dx}{dt} + \frac{\partial f}{\partial y} \cdot \frac{dy}{dt} + \frac{\partial f}{\partial z} \cdot \frac{dz}{dt} =$$

$$\nabla f(x(t), y(t), z(t)) \cdot \vec{r}\,'(t)$$

$$\int_a^b \frac{d}{dt} F(t)\, dt = F(b) - F(a)$$

Suppose $\vec{F}(x,y,z) = \vec{\nabla} f(x,y,z)$

$$\int_C \vec{F}(x,y,z) \cdot \vec{dr} = ?$$

$C: \vec{r}(t) = \langle x(t), y(t), z(t) \rangle$

$a \le t \le b$

$$\int_C \vec{F}(x,y,z) \cdot \vec{dr}$$

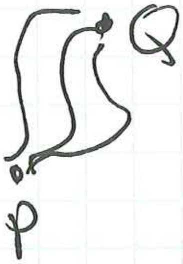
$$= \int_a^b \vec{F}(x(t), y(t), z(t)) \cdot \vec{r}\,'(t)\, dt$$

$$= \int_a^b (\nabla f)(x(t), y(t), z(t)) \cdot \langle x'(t), y'(t), z'(t) \rangle\, dt$$

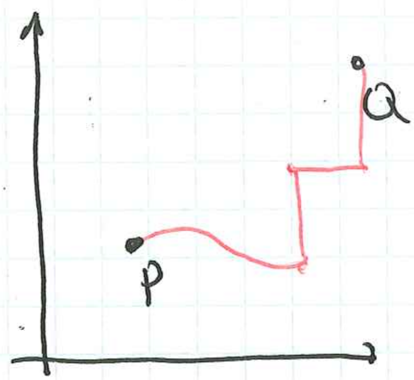
$$= \int_a^b \frac{d}{dt}\left[f(x(t), y(t), z(t)) \right] \cdot dt$$

$$= \int f(x(b), y(b), z(b)) - f(x(a), y(a), z(a))$$

If $\vec{F} = \nabla f$, and C is a path from P to Q,

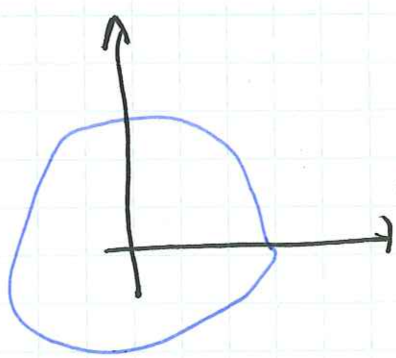


$$\int_C \vec{F} \cdot d\vec{r} = f(Q) - f(P)$$

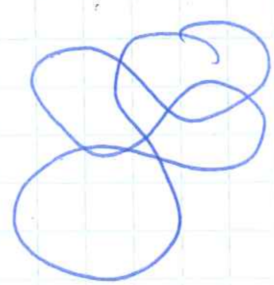


Piecewise smooth path

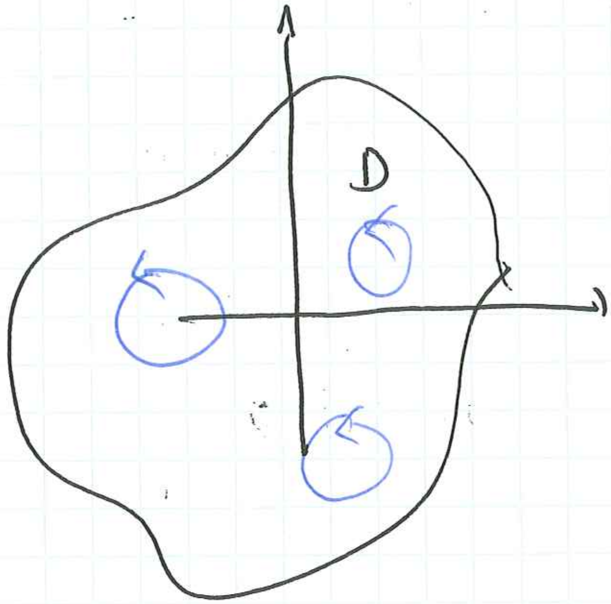
Simple Curve

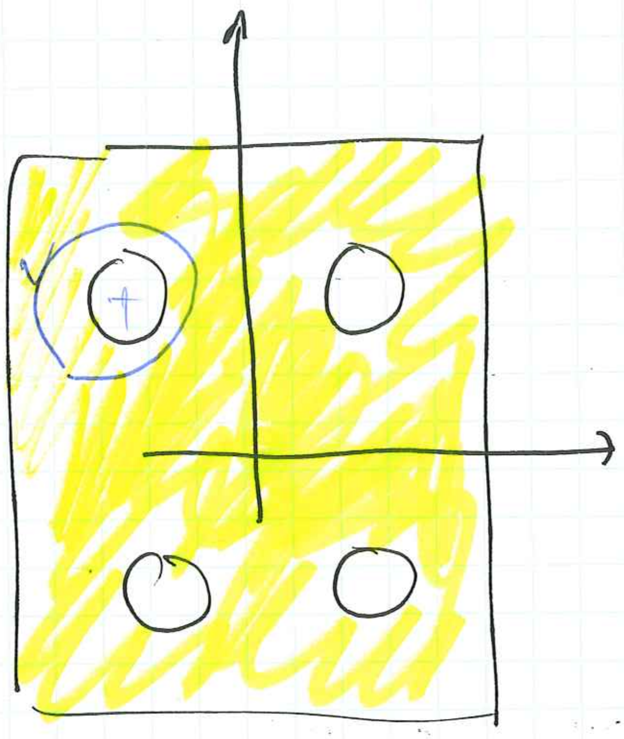


Non-simple curve



11/15/19 4





If $\vec{F}(x,y) = P(x,y)\,\hat{i} + Q(x,y)\,\hat{j}$

~~if a~~ is a gradient, $\vec{F} = \vec{\nabla} f$

then

$$P(x,y) = \frac{\partial f}{\partial x}(x,y)$$

$$Q(x,y) = \frac{\partial f}{\partial y}(x,y)$$

$$\left.\begin{aligned} \frac{\partial P}{\partial y}(x,y) &= \frac{\partial^2 f}{\partial y\,\partial x}(x,y) \\ \frac{\partial Q}{\partial x}(x,y) &= \frac{\partial^2 f}{\partial x\,\partial y}(x,y) \end{aligned}\right\} \text{same !!}$$

If $\vec{F} = P + iQ$ is a gradient vector field,

$$\boxed{\frac{\partial P}{\partial y} = \frac{\partial Q}{\partial x}}$$

11/15/19 6

			$\frac{\partial P}{\partial y}$	$\frac{\partial Q}{\partial x}$
1	$P(x,y) = -y$	$Q(x,y) = x$	-1	1
2	$P = x^2$	$Q = y^2$	0	0
3	$P = ye^x$	$Q = e^x + e^y$	e^x	e^x

~~4~~ $\frac{-y}{x^2+y^2}$ $\quad q = \frac{x}{x^2+y^2}$

$\frac{\partial}{\partial y}\left(\frac{-y}{x^2+y^2} \right.$

$C: \ x^2 + y^2 = 1$

$x(t) = \cos t$

$y(t) = \sin t$

$ds = dt$

$$\oint_C \vec{F} \cdot d\vec{r} = \int_0^{2\pi} \left\langle \frac{-\sin(t)}{1}, \frac{\cos(t)}{1} \right\rangle \cdot \langle -\sin t, \cos t \rangle \, dt$$

$\vec{F}$

$$= \int_0^{2\pi} (\sin^2 t + \cos^2 t)\, dt = 2\pi$$

$$\vec{F}(x,y) = \langle \underset{P}{\underset{\shortparallel}{y^2-2x}},\ \underset{Q}{\underset{\shortparallel}{2xy}} \rangle$$

$$\frac{\partial P}{\partial y} = 2y \qquad \frac{\partial Q}{\partial x} = 2y$$

$$\vec{F} = \vec{\nabla} f$$

1. $P(x,y) = \dfrac{\partial f}{\partial x}$ so

$$\frac{\partial f}{\partial x} = y^2 - 2x$$

$$f(x,y) = \int (y^2 - 2x)\, dx$$

$$= y^2 x - x^2 + C(y)$$

2. $$\frac{\partial f}{\partial y} = 2xy = (2xy + C'(y))$$

$$\therefore\ 0 = C'(y)$$

$$\therefore\ C(y) = C$$

$$\therefore\ f(x,y) = y^2x - x^2 + C$$

$$\vec{F} = \langle y^2 - 2x,\ 2xy \rangle$$