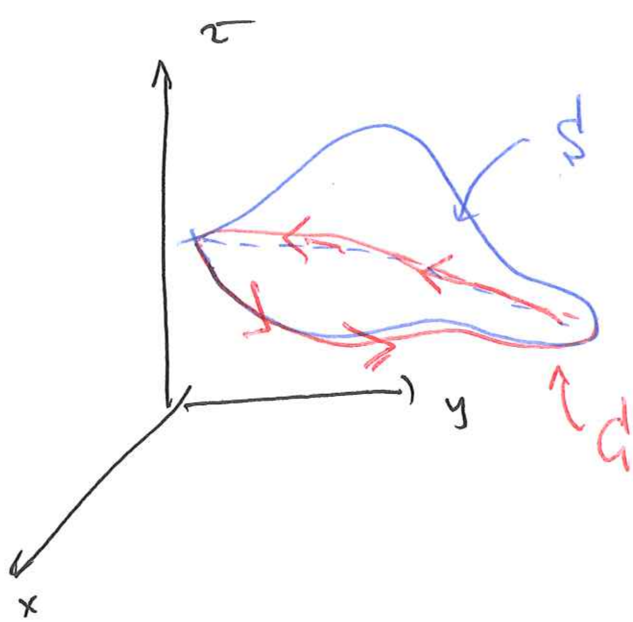
Office Hours Today

4 - 5 PM

755 POT

<u>Stokes</u>



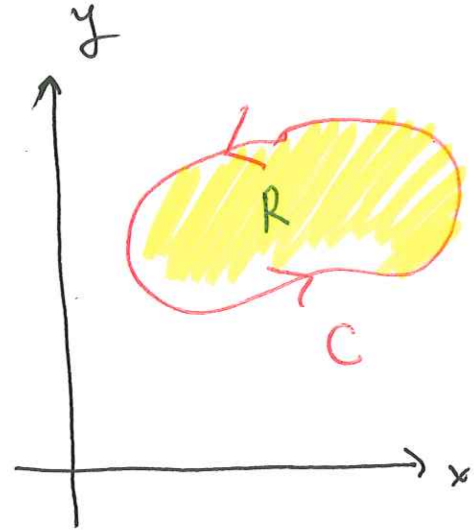
$\vec{F}(x,y,z) = P(x,y,z)\,\hat{i} +$

$Q(x,y,z)\,\hat{j} +$

$R(x,y,z)\,\hat{k}$

$$\int_C \vec{F}\cdot d\vec{r} = \iint_S \vec{\nabla}\times\vec{F}\cdot \underbrace{d\vec{S}}_{\hat{n}\,dS}$$

<u>Green</u>



$\vec{F}(x,y) = P(x,y)\,\hat{i} + Q(x,y)\,\hat{j}$

$$\int_C \vec{F}\cdot d\vec{r} =$$

$$\iint_R \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y}\right) dA$$

$$\int_C \vec{F} \cdot d\vec{r} =$$

$$\int_C P\,dx + Q\,dy$$

since

$$\int \vec{F}(x(t), y(t)) \cdot (x'(t)\,\hat{i} + y'(t)\,\hat{j}) =$$

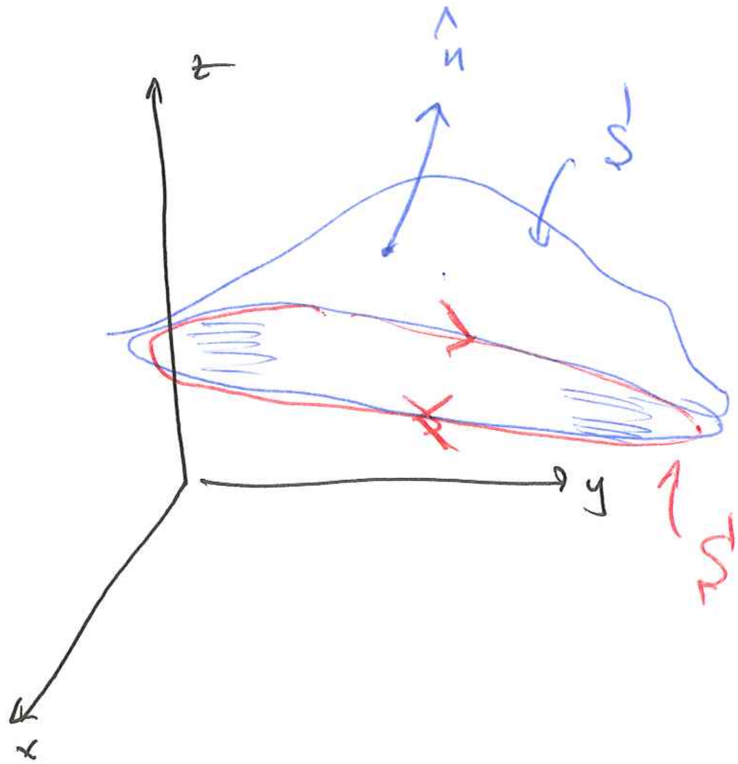
$$\int \left(P(x(t), y(t))\,\hat{i} + Q(x(t), y(t))\right) \cdot$$

$$\int (x'(t)\,\hat{i} + y'(t)\,\hat{j})\,dt =$$

$$\int P(x(t), y(t))\,x'(t)\,dt +$$

$$\int Q(x(t), y(t))\,y'(t)\,dt =$$

$$\int P\,dx + \int Q\,dy$$



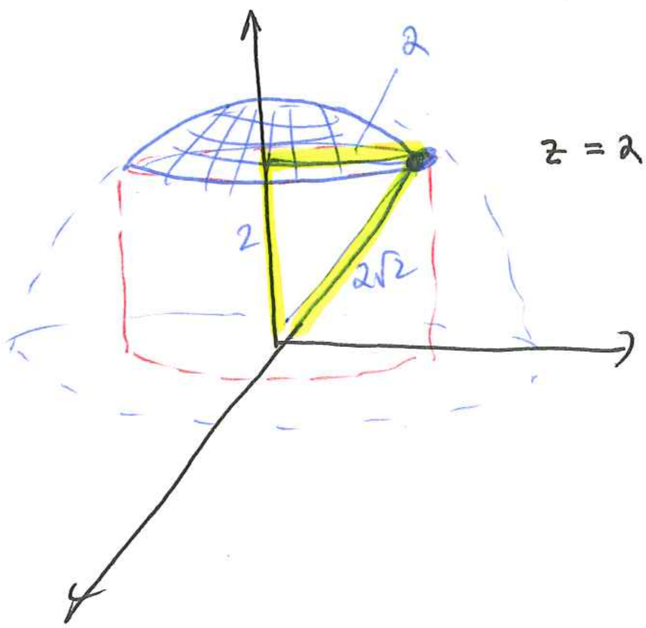
$$F = P\hat{i} + Q\hat{j} + R\hat{k}$$

$$\vec{\nabla} \times \vec{F} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ P & Q & R \end{vmatrix}$$

$$= \hat{i}\left(\frac{\partial R}{\partial y} - \frac{\partial Q}{\partial z}\right) +$$

$$\hat{j}\left(\frac{\partial P}{\partial z} - \frac{\partial R}{\partial x}\right) +$$

$$\hat{k}\left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y}\right)$$



$$x^2+y^2+z^2=8$$

$$x^2+y^2=4$$

Intersection:

$$\underbrace{x^2+y^2}_{4}+z^2=8$$

$$4+z^2=8$$

$$z^2=4$$

$$z=2$$

Circle of intersection:

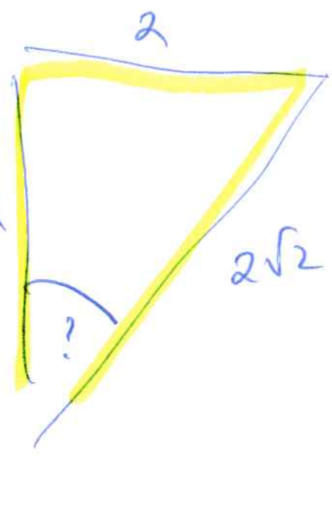
$$\begin{cases} z=2 \\ x^2+y^2=4 \end{cases}$$

Parameterize in spherical coords:

$$\rho = 2\sqrt{2}$$

$$0 \le \theta \le 2\pi$$

$$0 \le \varphi \le \pi/4$$



$$\vec{F}(x,y,z) = x^2 \sin z \, \hat{i} + y^2 \, \hat{j} + xy \, \hat{k}$$

C: Intersection of $z = 1 - x^2 - y^2$ and xy plane

$z = 0$

$\therefore \; 0 = 1 - x^2 - y^2$

$\therefore \; x^2 + y^2 = 1$

C: $\vec{r}(t) = \langle \cos t, \sin t, 0 \rangle$ $\quad 0 \le t \le 2\pi$

$$\int_C \vec{F} \cdot d\vec{r} = \int_0^{2\pi} \langle 0, \sin^2 t, \cos t \sin t \rangle \cdot \langle -\sin t, \cos t, 0 \rangle \, dt$$

$$= \int_0^{2\pi} \sin^2 t \cos t \, dt$$

$$= \frac{\sin^3 t}{3} \Big|_0^{2\pi} = 0$$

$\cancel{u = \sin t}$

$\cancel{du = \cos t \, dt}$

$u = \sin t$

$$\int \sin^2 t \cos t \, dt = \int u^2 \, du$$

$$= \frac{u^3}{3} + C$$

$$\int_C \vec{F} \cdot d\vec{r} =$$

$$\iint_S (\vec{\nabla} \times \vec{F}) \cdot d\vec{S}$$

$$\vec{\nabla} \times \vec{F} = \langle -2z, -2x, -2y \rangle$$

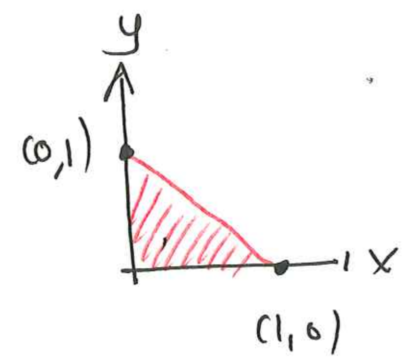
The surface S is part of the plane.

$$\boxed{x + y + z = 1}$$

that sits over

a triangle in the

xy plane.



$$\boxed{\begin{array}{l} 0 \le x \le 1 \\ 0 \le y \le 1 - x \end{array}}$$

Parameterize by

$$\vec{r}(x,y) = \langle x, y, 1 - x - y \rangle$$

for x, y range over the triangle.

$$\vec{r}_x(x,y) = \langle 1, 0, -1 \rangle$$

$$\vec{r}_y(x,y) = \langle 0, 1, -1 \rangle$$

$$\vec{r}_x \times \vec{r}_y = \begin{vmatrix} i & j & k \\ 1 & 0 & -1 \\ 0 & 1 & -1 \end{vmatrix}$$

$$= \hat{i} + \hat{j} + \hat{k}$$

$$= \langle 1, 1, 1 \rangle$$

S: lies over $\bar{W} = \{ (x,y) : 0 \le x \le 1,\ 0 \le y \le 1-x \}$

$$\vec{r}_x \times \vec{r}_y = \langle 1, 1, 1 \rangle$$

$$\vec{\nabla} \times \vec{F} = \langle -2z, -2x, -2y \rangle \qquad z = 1-x-y$$

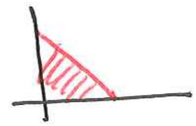
$$= \langle -2(1-x-y), -2x, -2y \rangle$$

$$\iint_S \vec{\nabla} \times \vec{F} \cdot d\vec{S} =$$

$$\int_0^1 \left(\int_0^{1-x} \langle -2(1-x-y), -2x, -2y \rangle \cdot \langle 1, 1, 1 \rangle \, dy \right) dx$$

$$= \int_0^1 \left(\int_0^{1-x} \left(-2(1-x-y) - 2x - 2y \right) dy \right) dx$$

$$= \int_0^1 \left(\int_0^{1-x} -2 \, dy \right) dx$$

$$= -2 \cdot \frac{1}{2}$$ area of 

$$= -1$$