# Math 213 - Points in Space 

Peter Perry

August 21, 2023

## First-Day Information

Please read through the online syllabus! The online calendar tells all.

- Online Text
- Webwork (Always Log in from Canvas!)
- 10 Quizzes
- Three Midterm Exams
- One Final Exam
- Class Participation

My Office Hours: MWF 2:00-3:00, 755 POT
My E-Mail: pperr0@uky.edu
In an urgent situation: (859) 361-7725

## Unit A: Vectors, Curves, and Surfaces

- August 21 - Points
- August 23 - Vectors
- August 25 - Dot Product
- August 28 - Cross Product
- August 30 - Equations of Planes
- September 1 - Equations of Lines
- September 6 - Curves
- September 8 - Integrating Along Curves
- September 11 - Integrating Along Curves
- September 13-Sketching Surfaces
- September 15 - Cylinders and Quadric Surfaces


## Points in the Plane



Points in the $x y$ plane are described by two coordinates

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$$
P_{2}\left(x_{2}, y_{2}\right)
$$

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The distance between two points is given by the Pythagorean Theorem:

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c^{2}=a^{2}+b^{2}
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$$

where

$$
\begin{aligned}
& a=x_{2}-x_{1} \\
& b=y_{2}-y_{1}
\end{aligned}
$$

## Points in the Plane

$$
\begin{aligned}
& \text { The distance between two points } \\
& \text { is given by the Pythagorean } \\
& \text { Theorem: } \\
& c^{2}=a^{2}+b^{2} \\
& \text { where } \\
& a=x_{2}-x_{1} \\
& b=y_{2}-y_{1} \\
& c^{2}=\left(x_{2}-x_{1}\right)^{2}+\left(y_{2}-y_{1}\right)^{2} \\
& d\left(P_{1}, P_{2}\right)=\sqrt{\left(x_{2}-x_{1}\right)^{2}+\left(y_{2}-y_{1}\right)^{2}}
\end{aligned}
$$

## Points in Space



Points in space have $(x, y, z)$ coordinates

## Points in Space



The point

$$
P=(1,2,3)
$$

is located as follows:
(1) Locate the point $(1,2)$ in the $x y$ plane
(2) Move up 3 units in the $z$ direction

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## Planes in Space



The $x y$ plane is the plane with $z=0$

## Planes in Space



The $x z$ plane is the plane
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## Planes in Space



The $y z$ plane is the plane with $x=0$

## Distances in Space



How do we find the distance from $P_{1}$ to $P_{2}$ ?

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a^{2}=\left(x_{2}-x_{1}\right)^{2}+\left(y_{2}-y_{1}\right)^{2}
$$

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\end{aligned}
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## The Distance Formula

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& c^{2}=a^{2}+b^{2} \\
& a^{2}=\left(x_{2}-x_{1}\right)^{2}+\left(y_{2}-y_{1}\right)^{2} \\
& b^{2}=\left(z_{2}-z_{1}\right)^{2} \\
& c^{2}=\left(x_{2}-x_{1}\right)^{2}+\left(y_{2}-y_{1}\right)^{2} \\
& +\left(z_{2}-z_{1}\right)^{2}
\end{aligned}
$$

## The Distance Formula

$$
\underbrace{\substack{c \\ P_{1}\left(x_{1}, y_{1}, z_{1}\right)}}_{\substack{\sim}}
$$

## Puzzler \#1



Find the set of points $(x, y, z)$ that obey the equation

$$
x^{2}+y^{2}+z^{2}=25
$$

## Puzzler \#2



Find the set of points $(x, y, z)$ that obey the equation

$$
x^{2}+y^{2}+z^{2}-4 x-4 y=0
$$

Puzzler \#3


Find the set of points that obey the equation $x^{2}+y^{2}=1$

## Puzzler \#4



Find the set of points that satisfy the equation $x=y$

## Puzzler \# 5

Find the equation of a sphere if one of its diameters has endpoints $(1,0,1)$ and $(3,4,3)$.


## Puzzler \#6

Describe the set of points $(x, y, z) \in \mathbb{R}^{3}$ that obey the inequality

$$
x^{2}+y^{2}+z^{2}<2 x-2 y+8
$$

## Extra-Credit Puzzler

Find the set of points that satisfy both of the equations

$$
\begin{array}{r}
x^{2}+y^{2}+z^{2}=6 \\
x^{2}+y^{2}=2
\end{array}
$$

## Reminders for the Week of August 21-25

- Tuesday 8/22 - Recitation on CLP 3 1.1-Points
- Wednesday 8/23 - Read CLP3 1.2 on Vectors before class
- Thursday 8/24 - Recitation on CLP3 1.2-Vectors
- Friday 8/25-Read CLP3 1.2 on Dot Products before class
- Friday 8/25 - Webwork A1 due at 11:59 PM

