Math 213 - Graphing Surfaces

Peter Perry

September 13, 2023

Unit A: Vectors, Curves, and Surfaces

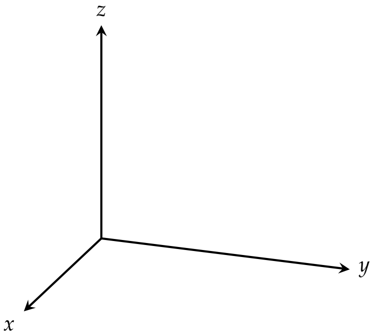
- August 21 - Points
- August 23 - Vectors
- August 25 - Dot Product
- August 28 - Cross Product
- August 30 - Equations of Planes
- September 1 - Equations of Lines
- September 6 - Curves
- September 8 - Integrating Along Curves
- September 11 - Integrating Along Curves
- September 13 - Sketching Surfaces
- September 15 - Cylinders and Quadric Surfaces

Exam 1

- Accommodation letters are due today by 5 PM
- Request for alternate exams are due Friday by 5 PM
- The exam will take place on Wednesday, September 20, 5:00-7:00 PM. You may bring one sheet of notes with formulas, definitions, etc., but NO solved problems. Be sure to bring your student ID. See the course syllabus for calculator policy..
- Your exams will be held in the following rooms:

Section 011	CP 139
Sections 012-014	CP 153

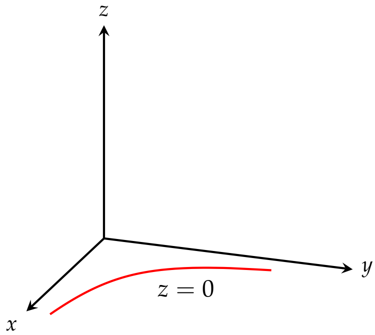
Finding a Surface by its Traces



Graph the surface $xy = 1$

- z can take any value

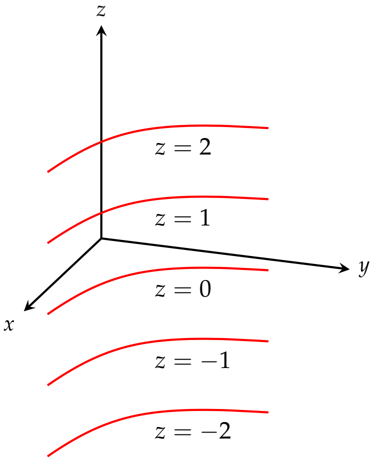
Finding a Surface by its Traces



Graph the surface $xy = 1$

- z can take any value
- We can look at the trace of the surface in the plane $z = 0$

Finding a Surface by its Traces



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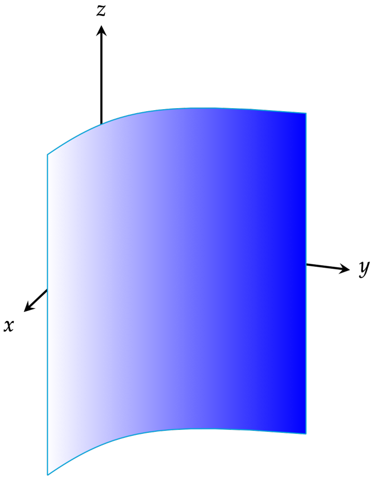
- We can shift the trace up or down to get traces in other planes

Finding a Surface by its Traces

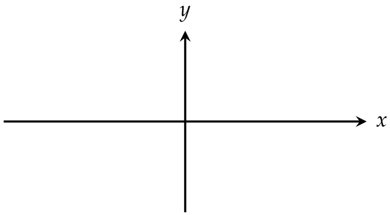


Graph the surface $xy = 1$

- z can take any value
- We can look at the trace of the surface in the plane $z = 0$
- We can shift the trace up or down to get traces in other planes

The surface $xy = 1$ is an example of a *cylinder*

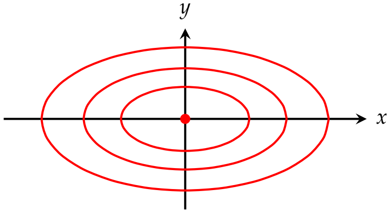
Level Curves



For a function $f(x,y)$ and a real number c, the *level curve* of $f(x,y)$ at c is the set of all points (x,y) with $f(x,y) = c$

Example: Find the level curves of $f(x,y) = x^2 + 4y^2$ for $c = 0, 2, 5, 10$

Level Curves



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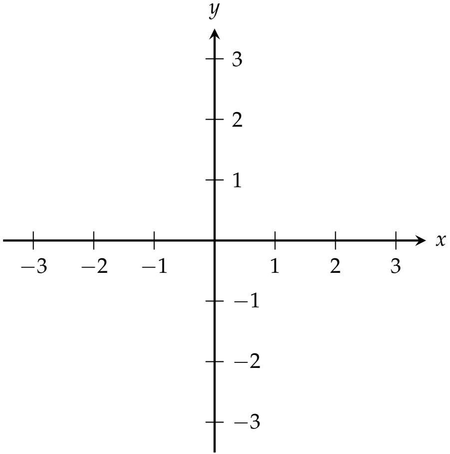
Example: Find the level curves of $f(x, y) = x^2 + 4y^2$ for $c = 0, 2, 5, 10$

This collection of level curves gives a *contour map* of the function $f(x, y)$

Puzzler #1

Sketch the level curves of $f(x, y) = xy$ for $c = 1, 2, 3$ and $c = -1, -2, -3$



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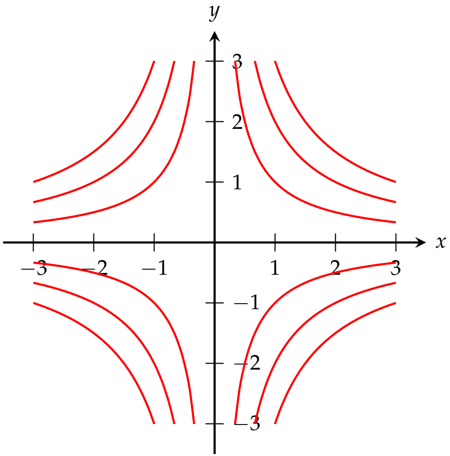
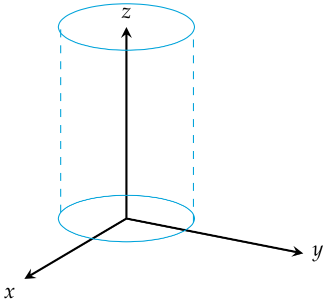


Level Surfaces

A *level surface* of a function $f(x,y,z)$ at c is the set of all points (x,y,z) so that $f(x,y,z) = c$



Find the level surfaces of the function

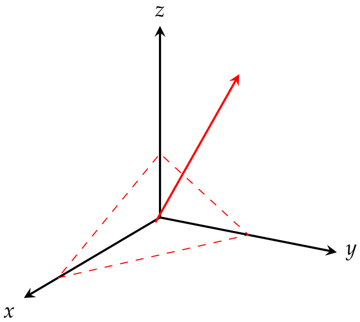
$$f(x,y,z) = x^2 + y^2$$

for $c = 1, 4, 9$

The level surfaces are cylinders with axis of symmetry on the z-axis

Puzzler #2

A *level surface* of a function $f(x, y, z)$ at c is the set of all points (x, y, z) so that $f(x, y, z) = c$



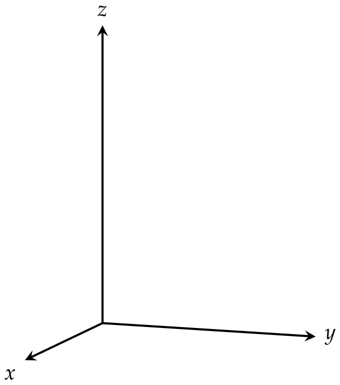
Find the level surfaces of the function

$$f(x, y, z) = x + 2y + 3z$$

for $c = 1, 2, 3$

The level surfaces are planes $x + 2y + 3z = c$ with normal vector $\mathbf{n} = \langle 1, 2, 3 \rangle$

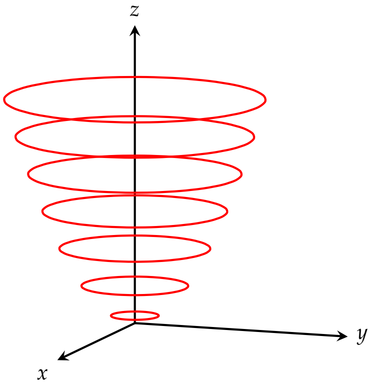
Graphs of Functions of Two Variables



Sketch the graph of

$$f(x,y) = x^2 + y^2$$

Graphs of Functions of Two Variables



Sketch the graph of

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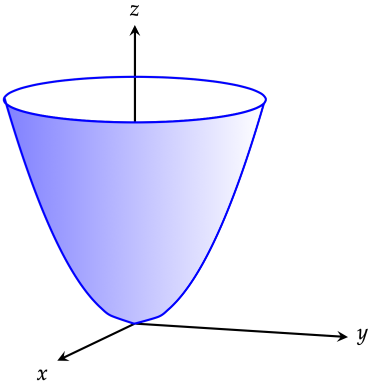
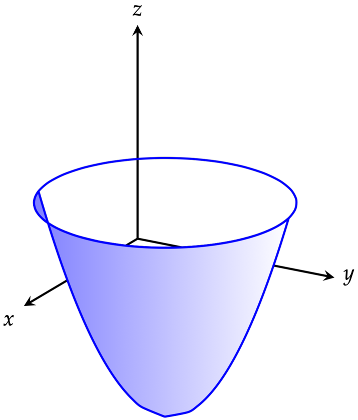
Graphs of Functions of Two Variables



Sketch the graph of

$$f(x,y) = x^2 + y^2$$

Puzzler #3



Sketch the graph of the function

$$f(x, y) = x^2 + y^2 - 2x - 2y$$

By completing the square we can write

$$f(x, y) = (x-1)^2 + (y-1)^2 - 2$$

so the vertex is shifted to $(1, 1, -2)$

Preview-Quadric Surfaces

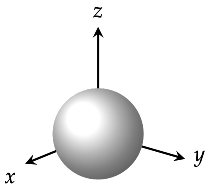
A *quadric surface* is any surface in the *xyz* plane defined by a quadratic equation

$$Ax^2 + By^2 + Cz^2 + Dxy + Exz + Fyz + Gx + Hy + Iz = L$$

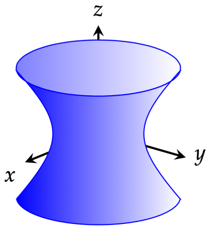
These equations can be reduced rotations and translations of coordinates to

$$Ax^2 + By^2 + Cz^2 = D$$

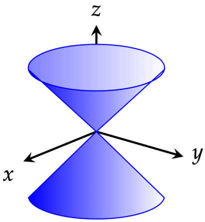
Here are some examples:



$x^2 + y^2 + z^2 = 9$



$x^2 + y^2 - z^2 = 1$



$x^2 + y^2 = z^2$

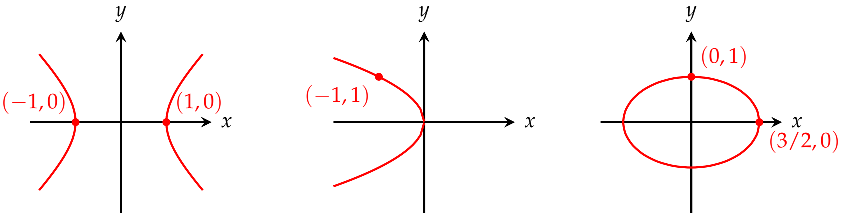
Preview-Quadric Surfaces

We can understand quadric surfaces by looking at their cross sections (traces) in planes $z = h$

Cross sections for quadric surfaces in the form $Ax^2 + By^2 + Cz^2 = D$ take one of the following forms:

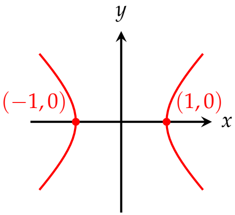
- $\alpha x^2 + \beta y^2 = \gamma$ with $\alpha, \beta, \gamma > 0$ (ellipse or circle)
- $\alpha x^2 - \beta y^2 = \gamma$ with $\alpha, \beta > 0$, $\gamma \neq 0$ (hyperbola) or $\gamma = 0$ (two lines)
- $x^2 = \delta y$ for $\delta \neq 0$ (parabola) or $\delta = 0$ (straight line)

Mix and Match

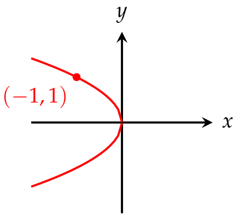


Preview-Quadric Surfaces

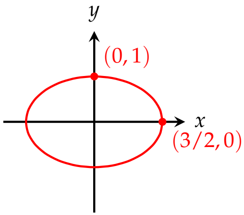
Every quadric surface is made by "stacking" these three basic kinds of curves



Hyperbola



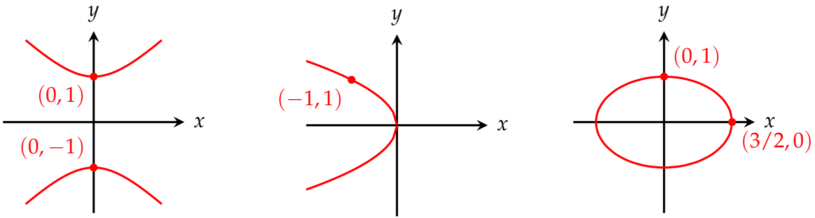
Parabola



Circle or Ellipse

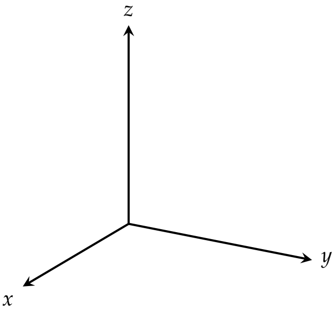
Preview-Quadric Surfaces

Every quadric surface is made by “stacking” these three basic kinds of curves



Hyperbola   Parabola   Circle or Ellipse

Sketching a Surface by Traces



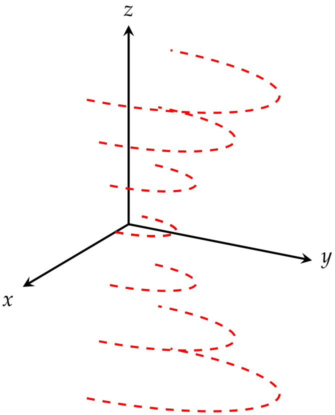
Problem: Sketch the surface

$$4x^2 + y^2 - z^2 = 1$$

- Find traces in planes $z = 0, \pm 1, \pm 2, \pm 3$
- Find traces in planes $x = 0, y = 0$

We'll cover this and other material about surfaces on Wednesda 9/13

Sketching a Surface by Traces



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$$4x^2 + y^2 - z^2 = 1$$

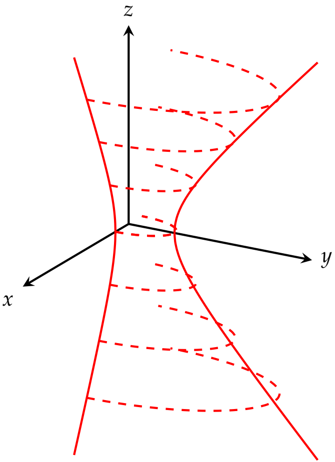
- Find traces in planes $z = 0, \pm 1, \pm 2, \pm 3$
- Find traces in planes $x = 0$, $y = 0$

$$4x^2 + y^2 = 1 + z^2$$

For each z, we get the equation of an ellipse

We'll cover this and other material about surfaces on Wednesda 9/13

Sketching a Surface by Traces



Problem: Sketch the surface

$$4x^2 + y^2 - z^2 = 1$$

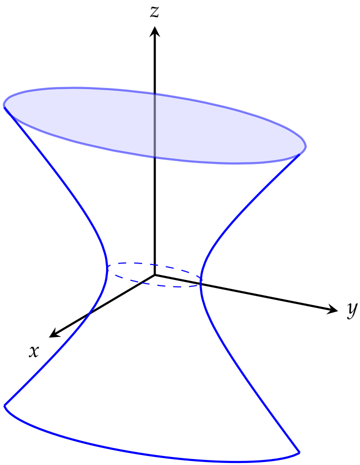
- Find traces in planes $z = 0, \pm 1, \pm 2, \pm 3$
- Find traces in planes $x = 0$, $y = 0$

In the plane $x = 0$, we get $y^2 - z^2 = 1$

In the plane $y = 0$, we get $4x^2 - z^2 = 1$

We'll cover this and other material about surfaces on Wednesda 9/13

Sketching a Surface by Traces

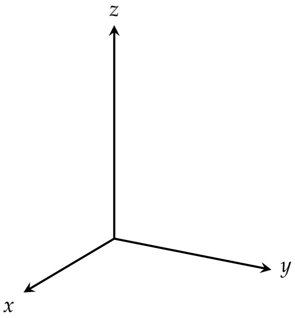


Here is the surface

$$4x^2 + y^2 - z^2 = 1$$

Note that the traces in planes parallel to the xy are ellipses and the traces in the xz and yz planes are hyperbolas

What Happens if One Sign Changes?

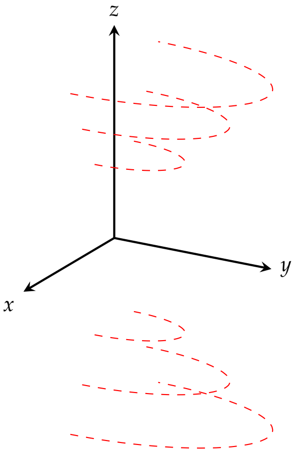


Let's try the same analysis with the surface

$$4x^2 + y^2 - z^2 = -1$$

- Find the traces in planes $z = 0, \pm 1, \pm 2, \pm 3, \pm 4$
- Find the traces in the xz and yz planes

What Happens if One Sign Changes?



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Traces in planes $z = 0, \pm 1, \pm 2, \pm 3, \pm 4$:

$$4x^2 + y^2 = z^2 - 1$$

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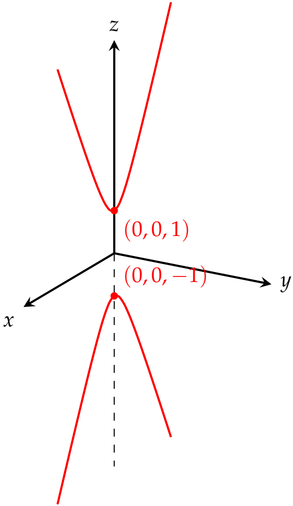


Trace in xz plane

Let's try the same analysis with the surface

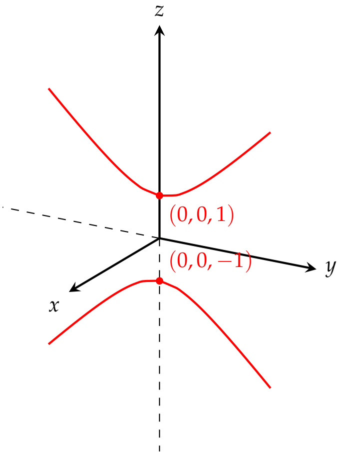
$$4x^2 + y^2 - z^2 = -1$$

- Find the traces in planes $z = 0, \pm 1, \pm 2, \pm 3, \pm 4$
- Find the traces in the xz and yz planes

In the xz plane, $4x^2 - z^2 = -1$

In the yz plane, $y^2 - z^2 = 1$

What Happens if One Sign Changes?



Trace in yz plane

Let's try the same analysis with the surface

$$4x^2 + y^2 - z^2 = -1$$

- Find the traces in planes $z = 0, \pm 1, \pm 2, \pm 3, \pm 4$
- Find the traces in the xz and yz planes

Reminders for the Week of September 11-15

- Homework A5 is due tonight at 11:59 PM
- Quiz #3 on curves and reparametrizations is due on Thursday at 11:59 PM
- Read CLP 3-1.8 and CLP 3-1.9 for Friday. If you haven't already looked at the Gallery of Quadric Surfaces yet, go here now!
- Exam #1 is next Wednesday, September 20. I need accomodation letters today by 5 PM and alternate exam requests by Friday September 15 at 5 PM!