Math 213 - Cylinders and Quadric Surfaces

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September 17, 2023

Unit A: Vectors, Curves, and Surfaces

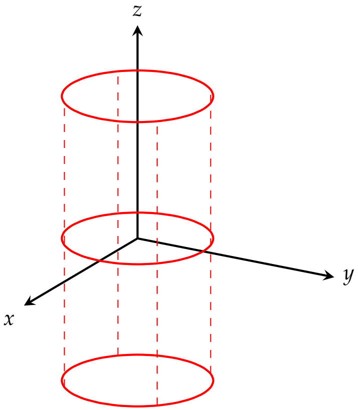
- August 21 - Points
- August 23 - Vectors
- August 25 - Dot Product
- August 28 - Cross Product
- August 30 - Equations of Planes
- September 1 - Equations of Lines
- September 6 - Curves
- September 8 - Integrating Along Curves
- September 11 - Integrating Along Curves
- September 13 - Sketching Surfaces
- September 15 - Cylinders and Quadric Surfaces

Introducing the Orangutan

The name "orangutan" is derived from the Malay words orang, meaning "person", and hutan, meaning "forest"

Source: Wikipedia
Image: PAP

Cylinders



A *cylinder* is a surface consisting of all lines

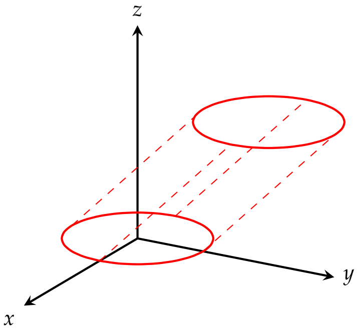
- parallel to a given line
- pass through a given fixed curve

Here are some examples. What are the given line and the given curve?

Example 1:

$$x^2 + y^2 = 1$$

Cylinders



A *cylinder* is a surface consisting of all lines

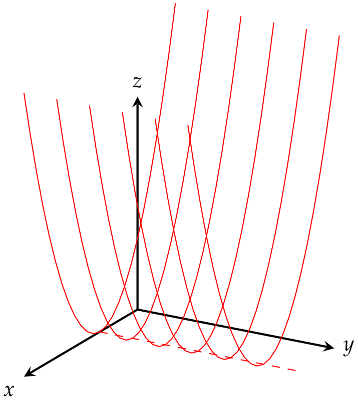
- parallel to a given line
- pass through a given fixed curve

Here are some examples. What are the given line and the given curve?

Example 2:

$$x^2 + (y - z)^2 = 1$$

Cylinders



A *cylinder* is a surface consisting of all lines

- parallel to a given line
- pass through a given fixed curve

Here are some examples. What are the given line and the given curve?

Example 3:

$$z = (x-1)^2$$

Quadric Surfaces

A *quadric surface* is the set of all points satisfying an equation of the form

$$Ax^2 + By^2 + Cz^2 + Dx + Ey + Fz = G$$

for some constants A, B, C, D, E, F, and G.

By completing the square we can reduce the equation above to

$$A(x-a)^2 + B(y-b)^2 + C(z-c)^2 = H$$

for new constants a, b, c, and H.

By moving the point (a, b, c) to the origin $(0, 0, 0)$ we get the equation

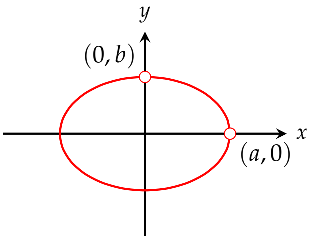
$$Ax^2 + By^2 + Cz^2 = L$$

for a new constant L.

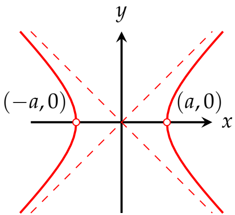
You get very different surfaces depending on the signs of A, B, C, and L.

Time Out: Conic Sections

Remember these *conic sections*:

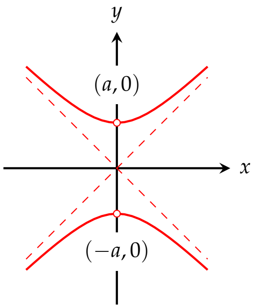


Ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

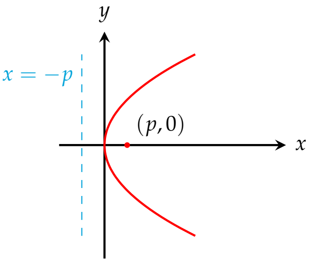


Hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$

Time Out: Conic Sections

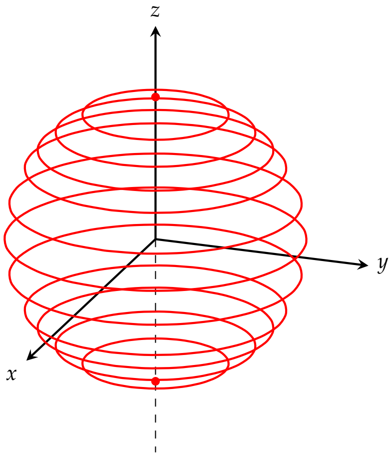


Hyperbola $\frac{y^2}{a^2} - \frac{x^2}{b^2} = 1$



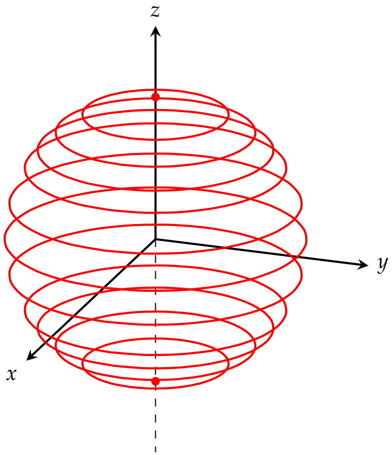
Parabola $y^2 = 4px$

A Gallery of Quadric Surfaces: Part I



Example 1: $x^2 + y^2 + z^2 = r^2$

A Gallery of Quadric Surfaces: Part I



Example 1: $x^2 + y^2 + z^2 = r^2$

Traces in $z = h$:

$$x^2 + y^2 + h^2 = r^2$$

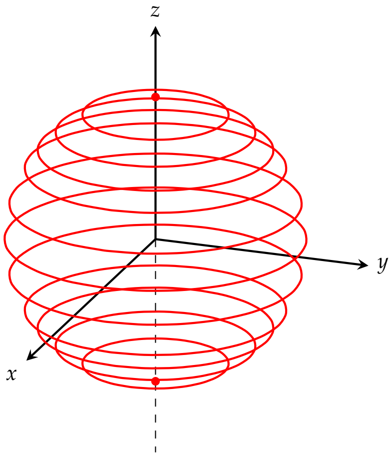
or

$$x^2 + y^2 = r^2 - h^2$$

The traces are circles of radius $\sqrt{r^2 - h^2}$

Note that there is *no trace* if $|h| > r$!

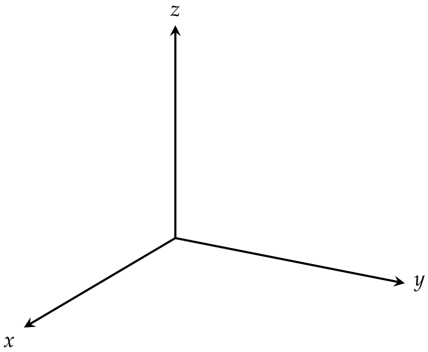
A Gallery of Quadric Surfaces: Part I



Example 1: $x^2 + y^2 + z^2 = r^2$

This quadric surface is a **sphere** of radius r with center $(0, 0, 0)$

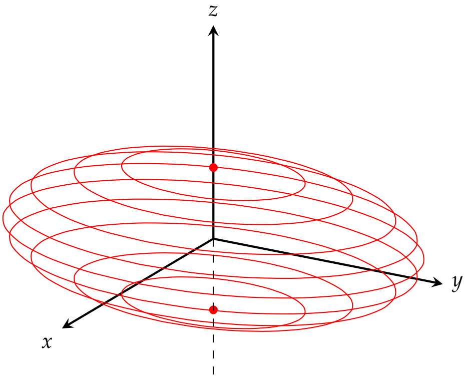
A Gallery of Quadric Surfaces: Part II



$$\frac{x^2}{4} + \frac{y^2}{9} + z^2 = 1$$

Traces in planes $z = h$:

A Gallery of Quadric Surfaces: Part II



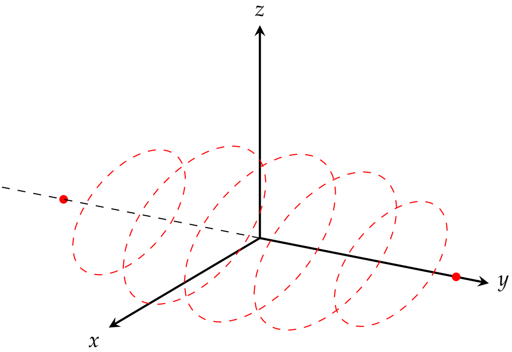
$$\frac{x^2}{4} + \frac{y^2}{9} + z^2 = 1$$

Traces in planes $z = h$:

$$\frac{x^2}{4} + \frac{y^2}{9} = 1 - h^2$$

Are there any values of h for which there is *no* trace?

A Gallery of Quadric Surfaces: Part II



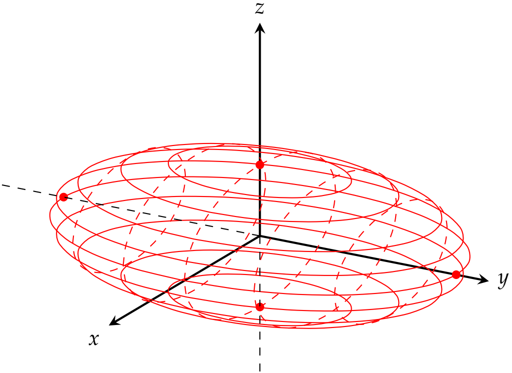
$$\frac{x^2}{4} + \frac{y^2}{9} + z^2 = 1$$

Traces in planes $y = h$:

$$\frac{x^2}{4} + z^2 = 1 - \frac{h^2}{9}$$

Are there any values of h for which there is *no* trace?

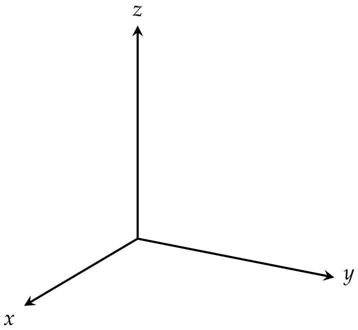
A Gallery of Quadric Surfaces: Part II



$$\frac{x^2}{4} + \frac{y^2}{9} + z^2 = 1$$

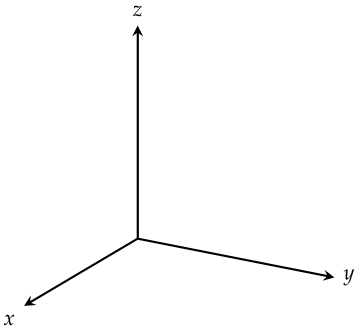
This quadric surface is an **ellipsoid**

A Gallery of Quadric Surfaces: Part III



$$z^2 = x^2 + y^2$$

A Gallery of Quadric Surfaces: Part III

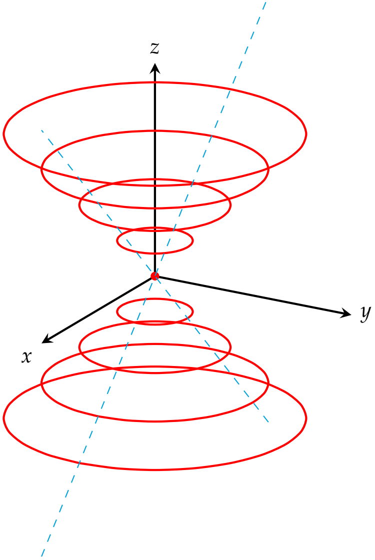


$$z^2 = x^2 + y^2$$

What are the traces in planes $z = h$?

What are the traces in the xz and yz planes?

A Gallery of Quadric Surfaces: Part III



$$z^2 = x^2 + y^2$$

What are the traces in planes $z = h$?

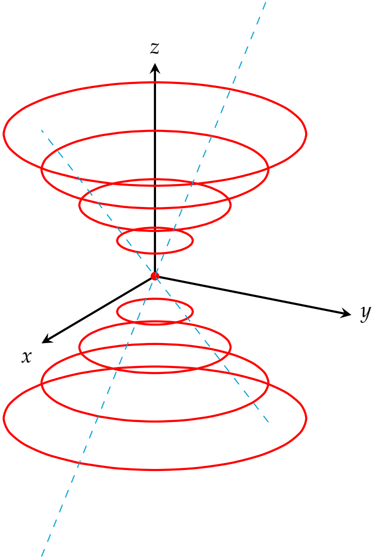
What are the traces in the xz and yz planes?

Planes $z = h$:

$$x^2 + y^2 = h^2$$

xz plane: $z^2 = x^2$
yz plane: $z^2 = y^2$

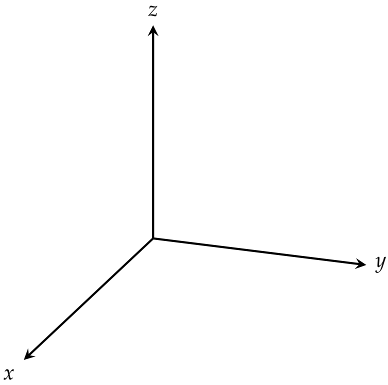
A Gallery of Quadric Surfaces: Part III



$$z^2 = x^2 + y^2$$

This quadric surface is a **cone**

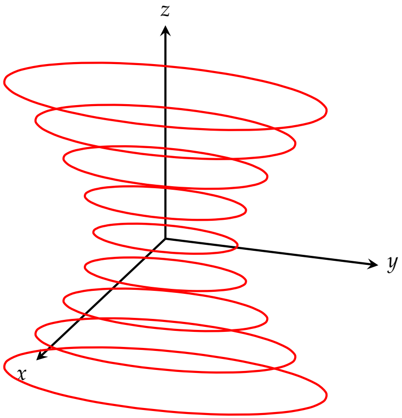
A Gallery of Quadric Surfaces: Part IV



$$4x^2 + y^2 - z^2 = 1$$

What are its traces in the planes $z = h$?

A Gallery of Quadric Surfaces: Part IV



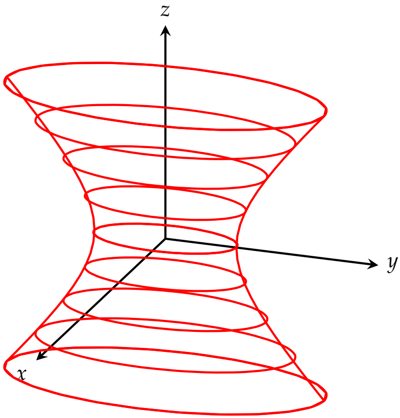
$$4x^2 + y^2 - z^2 = 1$$

What are its traces in the planes $z = h$?

$$4x^2 + y^2 = 1 + h^2$$

Are there any values of h for which there is *no* trace?

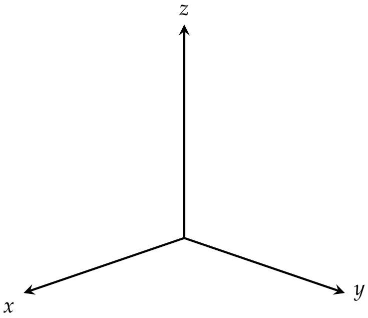
A Gallery of Quadric Surfaces: Part IV



$$4x^2 + y^2 - z^2 = 1$$

This surface is a **hyperboloid of one sheet**

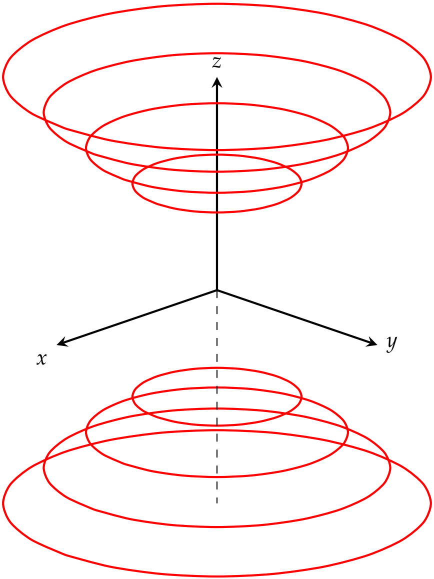
A Gallery of Quadric Surfaces: Part V



$$x^2 + y^2 - z^2 = -1$$

What are its traces in planes parallel to the *xy* plane?

A Gallery of Quadric Surfaces: Part V



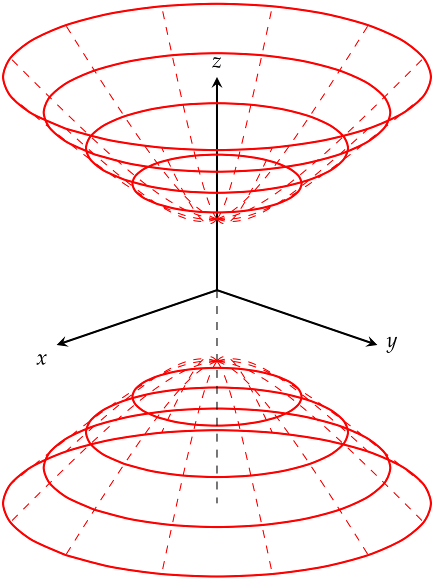
$$x^2 + y^2 - z^2 = -1$$

What are its traces in planes parallel to the xy plane?

$$x^2 + y^2 = h^2 - 1$$

Are there any values of h for which there is *no trace*?

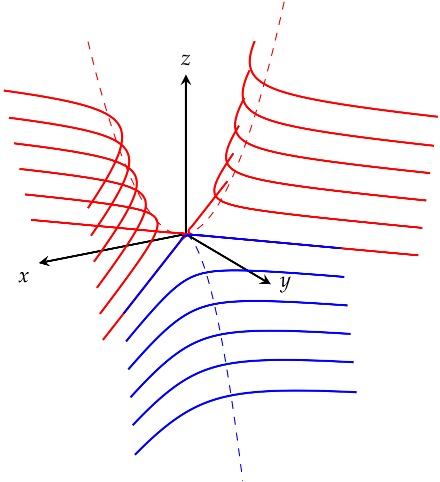
A Gallery of Quadric Surfaces: Part V



$$x^2 + y^2 - z^2 = -1$$

This quadric surface is a **hyperboloid of two sheets**

Mystery Surface



$$z = x^2 - y^2$$

Shown are traces in planes $z = h$ for $h > 0$ (red) and $h < 0$ (blue).

What is this surface?

Reminders for the Week of September 11-15

- Alternate exam requests are due by 5 PM tonight
- Homework A6 is due Monday 11:59 PM!
- Your first exam is on Wednesday September 20 at 5:00 PM
- Section 11 takes the exam in CP 139
- Sections 12-14 takes the exam in CP 153