Math 213 - Triple Integrals

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October 20, 2023

Unit C: Multiple Integrals

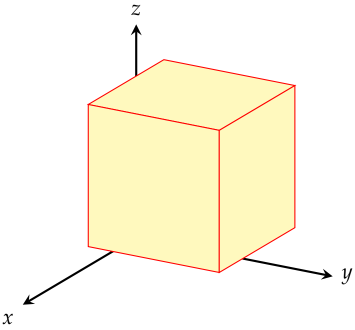
- October 13 - Double Integrals
- October 16 - Double Integrals in Polar Coordinates
- October 20 - Triple Integrals
- October 25 - Triple Integrals, Cylindrical Coordinates
- October 27 - Triple Integrals, Spherical Coordinates
- October 30 - Triple Integrals, General Coordinates
- November 1 - Vector Fields
- November 3 - Conservative Vector Fields
- November 6 - Line integrals
- November 8 - Parametrized Surfaces
- November 10 - Tangent Planes to Surfaces
- November 13 - Surface Integrals
- November 15 - Exam III Review

Just Like Double Integrals, Only More So

The triple integral of a function $f(x, y, z)$ over a region $\mathcal{R}$ of three-dimensional space is

$$\iiint_{\mathcal{R}} f(x, y, z)\, dV$$

Simplest case: $\mathcal{R}$ is a box $[a, b] \times [c, d] \times [e, f]$



$$\iiint_{\mathcal{R}} f(x, y, z)\, dV = \int_a^b \int_c^d \int_e^f f(x, y, z)\, dz\, dy\, dx$$

That is, a triple integral can be written as a "triple iterated integral" and evaluated from the inside out.

Puzzler #1

$$\iiint_{\mathcal{R}} f(x,y,z)\,dV = \int_a^b \int_c^d \int_e^f f(x,y,z)\,dz\,dy\,dx$$

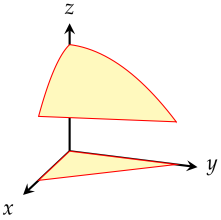
Find $\iiint_{\mathcal{R}} xyz\,dV$ if $\mathcal{R}$ is the region

$$1 \le x \le 3,\ 2 \le y \le 4,\ 0 \le z \le 2$$

More So

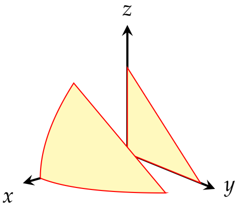
For triple integrals, there are (at least) three ways to set up a triple integral over a domain in *xyz* space:

Bottom to Top:



$$0 \leq z \leq h(x, y)$$

Back to Front:



$$0 \leq x \leq f(y, z)$$

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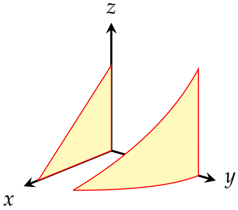
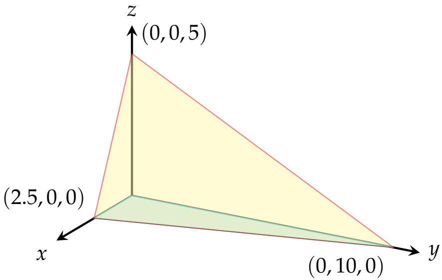
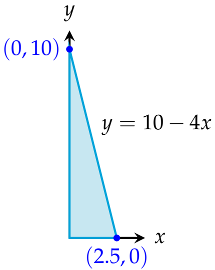


$$0 \leq y \leq r(x, z)$$

Another Triple Integral



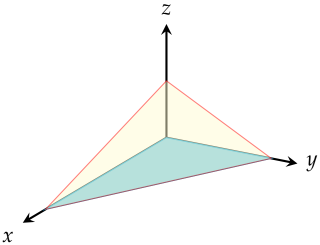


Find $\iiint_{\mathcal{R}} 6z^2\, dV$ if $\mathcal{R}$ is the region below the plane $4x + y + 2z = 10$ and in the first octant.

See Paul's Online Math Notes, §15.5 Practice Problems

Puzzler #2

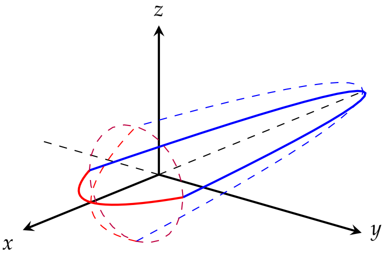
Evaluate $\iiint_{\mathcal{R}} 5x^2\,dV$ where $\mathcal{R}$ is the region below $x + 2y + 4z = 8$ in the first octant.



See Paul's Online Math Notes, §15.5, problem 5

Puzzler #3

Evaluate $\iiint_{\mathcal{R}} 20x^3\,dV$ if $\mathcal{R}$ is the region between $x = 2 - y^2 - z^2$ and $x = 5y^2 + 5z^2 - 6$.



Hint: These two bounding surfaces are paraboloids which intersect in a circle.

$x = 2 - y^2 - z^2$ is in red

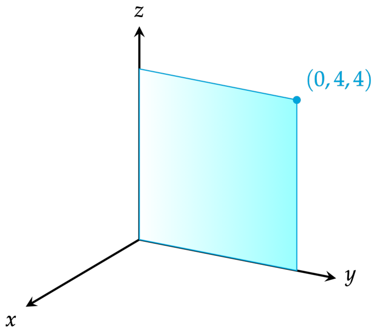
$x = 5y^2 + 5z^2 - 6$ is in blue

See Paul's Online Math Notes, §15.5, problem 10

Puzzler #4

Write $\iiint_{\mathcal{R}} f(x, y, z)\, dV$ as a triple iterated integral if $\mathcal{R}$ is the region bounded by the planes $x = 0$, $y = 0$, $z = 0$, $y = 4 - x$ and $z = 4 - x^2$

$x = 0$ (cyan)



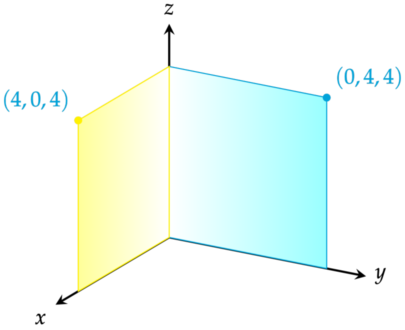
See CLP-3, section 3.5 Example 3.5.2

Puzzler #4

Write $\iiint_{\mathcal{R}} f(x,y,z)\,dV$ as a triple iterated integral if $\mathcal{R}$ is the region bounded by the planes $x=0$, $y=0$, $z=0$, $y=4-x$ and $z=4-x^2$

$x=0$ (cyan)

$y=0$ (yellow)



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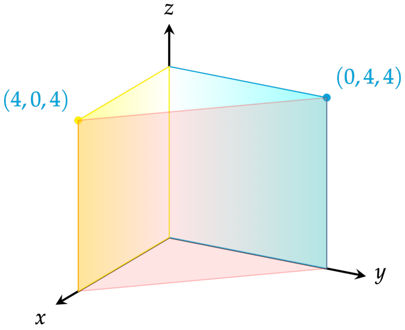

$x = 0$ (cyan)

$y = 0$ (yellow)

$y = 4 - x$ (red)

See CLP-3, section 3.5 Example 3.5.2

Puzzler #4

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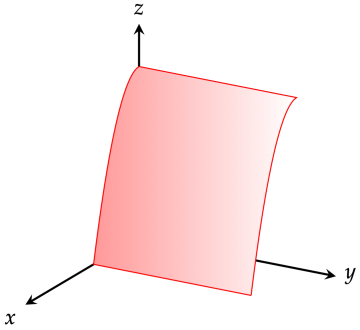

$z = 4 - x^2$

See CLP-3, section 3.5 Example 3.5.2

Puzzler #4

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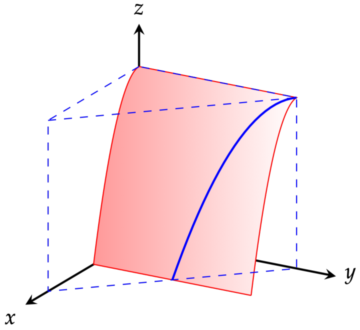

The plane $y = 4 - x$ and the surface $z = 4 - x^2$ intersect in the curve $(x, 4 - x, 4 - x^2)$

See CLP-3, section 3.5 Example 3.5.2

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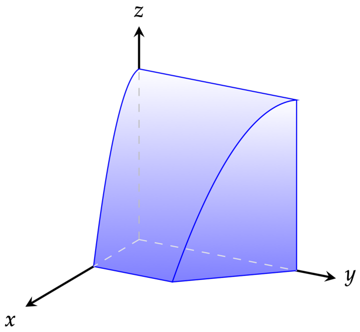

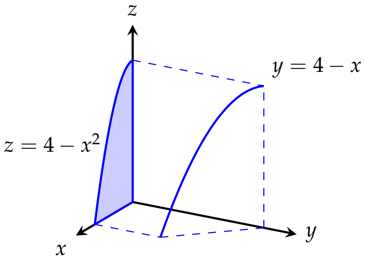

See CLP-3, section 3.5 Example 3.5.2

Reminders for the Week of October 22-26

- Fall Break, October 23-24
- Webwork B7 on Double Integrals due October 25
- Quiz #7 on Double Integrals due October 26
- Webwork B8 on Double Integrals in Polar Coordinates due October 27