Math 213 - Exam III Review

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Exam 3 Topics

Multiple Integrals

Double integrals
- Polar coordinates

Triple integrals
- Cylindrical coordinates
- Spherical coordinates

Change of Variables in multiple integrals

Vector Fields, Line Integrals

Vector fields
Conservative vector fields
Line integrals of scalar functions
Line integrals of vector fields

Basic Formulas

Coordinate Systems

Polar coordinates

$$r = \sqrt{x^2 + y^2} \qquad x = r\cos\theta$$
$$\theta = \arctan(y/x) \qquad y = r\sin\theta$$

Spherical coordinates

$$\rho = \sqrt{x^2 + y^2 + z^2}$$
$$\theta = \arctan(y/x)$$
$$\varphi = \arctan\left(\sqrt{x^2 + y^2}/z\right)$$

Integrals

Polar coordinates

$$\iint_A f(x, y)\, dA = \iint_D f(r\cos\theta, r\sin\theta)\, r\, dr\, d\theta$$

Cylindrical Coordinates

$$\iiint_R f(x, y, z)\, dV = \iiint f(r\cos\theta, r\sin\theta, z)\, r\, dz\, dr\, d\theta$$

Basic Formulas

Spherical Coordinates

$$\iiint_R f(x,y,z)\,dV = \iiint f(\rho\sin\theta\cos\varphi, \rho\sin\theta\sin\varphi, \rho\cos\varphi)\rho^2\sin\varphi\,d\rho\,d\theta,\,d\varphi$$

Change of variables

If $(x(u,v), y(u,v))$ maps a region U of the uv plane to a region D of the xy plane,

$$\iint_D f(x,y)\,dA = \iint_U f(x(u,v),y(u,v))\left|\frac{\partial(x,y)}{\partial(u,v)}\right|\,du\,dv$$

where

$$\frac{\partial(x,y)}{\partial(u,v)} = \begin{vmatrix} \frac{\partial x}{\partial u} & \frac{\partial x}{\partial v} \\ \frac{\partial y}{\partial u} & \frac{\partial y}{\partial v} \end{vmatrix}$$

Line Integrals

If $\mathcal{C}$ is a parameterized curve

$$\mathbf{r}(t) = \langle x(t), y(t), z(t) \rangle, \quad a \leq t \leq b,$$

the line integral of a function $f(x, y, z)$ over $\mathcal{C}$ is:

$$\int_{\mathcal{C}} f(x, y, z)\, ds = \int_a^b f(x(t), y(t), z(t)) \sqrt{x'(t)^2 + y'(t)^2 + z'(t)^2}\, dt$$

and the line integral of a vector field $\mathbf{F}(\mathbf{r})$ over $\mathcal{C}$ is

$$\int_{\mathcal{C}} \mathbf{F} \cdot d\mathbf{r} = \int_a^b \mathbf{F}(\mathbf{r}(t)) \cdot \mathbf{r}'(t)\, dt$$

Line Integrals

$$\int_C f(x,y,z)\,ds = \int_a^b f(x(t),y(t),z(t))|\mathbf{r}'(t)|\,dt$$

Find the line integral of $f(x,y,z) = 2x^2 + 8z$ over the curve $\mathbf{r}(t) = \langle e^t, t^2, t\rangle$ for $0 \leq t \leq 7$.

Line Integrals

Find $\int_C 4y\,dx + 4x\,dy + 3\,dz$ if

$$\mathbf{r}(t) = \langle 3\cos t, 3\sin t, 3t\rangle, \quad 0 \le t \le \pi.$$

(This is another way of writing $\int_C \mathbf{F}\cdot d\mathbf{r}$ if $\mathbf{F}(x,y,z) = \langle 4y, 4x, 3\rangle$ and $\mathcal{C}$ is the path above)

Triple Integrals

Set up but do not compute

$$\int_E xyz\, dV$$

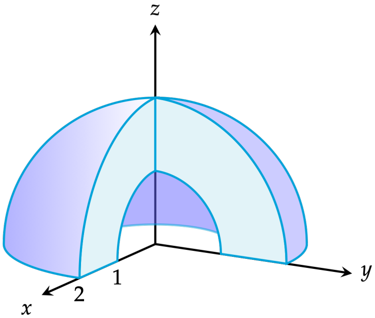
if E is the region in the first octant underneath the plane $3x + 6y + 3z = 9$.

Cylindrical Coordinates

Set up an integral in cylindrical coordinates for $\iiint_E z\,dV$, where E is enclosed by the paraboloid $z = x^2 + y^2$ and the plane $z = 4$

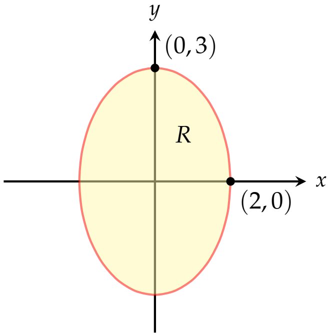
Spherical Coordinates

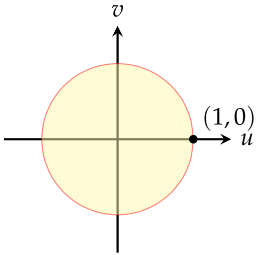
Set up a triple integral for a function $f(x, y, z)$ over the region shown below.



Change of Variables

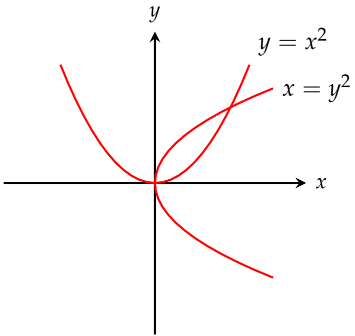
Find $\iint_R x^2 \, dA$ if R is the region bounded by the ellipse $9x^2 + 4y^2 = 36$ using the transformation $x = 2u$, $y = 3v$





Double Integrals

Find the volume of the region under the plane $3x + 2y - z = 0$ and above the region enclosed by the parabolas $y = x^2$ and $x = y^2$



Conservative Vector Fields

Find a potential for the vector field

$$\mathbf{F}(x,y) = (ye^x + \sin y)\mathbf{i} + (e^x + x\cos y)\mathbf{j}$$

and find its line integral along any path from $(0,0)$ to $(0,\pi/2)$.

Parting Words

Good luck on tonight's exam!