

## Quiz 5

Name: \_\_\_\_\_ Section and/or TA: \_\_\_\_\_

Answer all questions in a clear and concise manner. Unsupported answers will receive *no credit*.

1. (2 points) Use the Chain Rule to find  $\frac{dz}{dt}$  where  $z = x^2 + y^2 + xy$ ,  $x = \sin(t)$ , and  $y = e^t$ . No need to simplify.

**Solution:** Recall  $\frac{dz}{dt} = \frac{\partial z}{\partial x} \frac{dx}{dt} + \frac{\partial z}{\partial y} \frac{dy}{dt}$ . We find:  $\frac{\partial z}{\partial x} = 2x + y$ ,  $\frac{\partial z}{\partial y} = 2y + x$ ,  $\frac{dx}{dt} = \cos(t)$ , and  $\frac{dy}{dt} = e^t$ . Thus  $\frac{dz}{dt} = (2x + y)\cos(t) + (2y + x)e^t$ .

2. (3 points) The function  $f(x, y) = \frac{x}{x+y}$  is differentiable at the point  $(2, 1)$ . Find the linearization  $L(x, y)$  of the function at this point.

**Solution:** Computing the partial derivatives, we get

$$f_x(x, y) = \frac{y}{(x+y)^2}$$

$$f_y(x, y) = \frac{-x}{(x+y)^2}$$

Then we find the linearization

$$\begin{aligned}L(x, y) &= f_x(2, 1)(x - 2) + f_y(2, 1)(y - 1) + f(2, 1) \\ &= \frac{1}{9}(x - 2) - \frac{2}{9}(y - 1) + \frac{2}{3}\end{aligned}$$