Quiz 6

Name: ______________________ Section and/or TA: ______________

Answer all questions in a clear and concise manner. Unsupported answers will receive *no credit*.

1. (2 points) Let $f(x, y) = x^3 e^{xy}$.
   (a) (1 point) Find $\nabla f(2, 0)$.

   Solution: $\nabla f = \langle f_x(x, y), f_y(x, y)\rangle = \langle x^3 y e^{xy} + 3x^2 e^{xy}, x^4 e^{xy}\rangle$.
   $$\nabla f(2, 0) = \langle 12, 16\rangle$$

   (b) (1 point) Find the directional derivative of f at the point $(2, 0)$ in the direction $\theta = \frac{\pi}{4}$.

   Solution: In this case $\mathbf{u} = \langle \cos\frac{\pi}{4}, \sin\frac{\pi}{4}\rangle = \langle \frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\rangle$ so
   $$D_{\mathbf{u}} f(2, 0) = \langle 12, 16\rangle \cdot \left\langle \frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right\rangle = 6\sqrt{2} + 8\sqrt{2} = 14\sqrt{2}$$

2. (3 points) Find the critical point of $f(x, y) = xy - 2x - 2y - x^2 - y^2$ and use the Second Derivative Test to determine if it is a local maximum, local minimum, or a saddle point.

   Solution: $f_x = y - 2 - 2x = 0$ when $y = 2 + 2x$ and $f_y = x - 2 - 2y = 0$ when $x = 2 + 2y$. So $f_x = 0 = f_y$ when
   $$y = 2 + 2(2 + 2y) = 6 + 4y.$$
   Hence we have a critical point at $(-2, -2)$. Since
   $$f_{xx} = -2 \qquad f_{yy} = -2 \qquad f_{xy} = 1,$$
   we have $D(-2, -2) = (-2)(-2) - (1)^2 = 3 > 0$. So $D(-2, -2) > 0$ and $f_{xx} = -2 < 0$ implies $(-2, -2)$ is a local maximum.