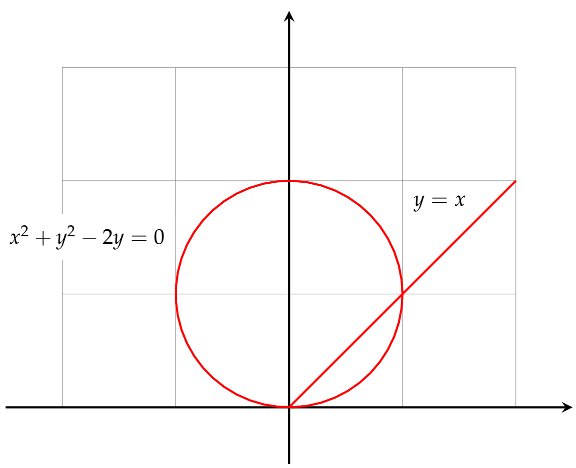
Quiz 7

Name: ______________________ Section and/or TA: ______________

Answer all questions in a clear and concise manner. Unsupported answers will receive *no credit*.

1. (3 points) Compute the area of the region D bounded above by the line $y = x$ and below by the circle $x^2 + y^2 - 2y = 0$ by converting to polar coordinates.



Solution: The polar form of the line $y = x$ is given by $\theta = \frac{\pi}{4}$, and the form for the circle is

$$x^2 + y^2 = 2y \Rightarrow r^2\cos^2(\theta) + r^2\sin^2(\theta) = 2r\sin(\theta)$$
$$\Rightarrow r^2 = 2r\sin(\theta) \Rightarrow r = 2\sin(\theta).$$

That is, the region D is determined by θ varying from 0 to $\frac{\pi}{4}$ while r varies from 0 to $2\sin(\theta)$. The area then is given by integral

$$A = \iint_D dA = \int_0^{\pi/4}\int_0^{2\sin(\theta)} r\, dr d\theta$$
$$= \int_0^{\pi/4} \frac{1}{2}r^2\Big|_{r=0}^{r=2\sin(\theta)} d\theta$$
$$= \int_0^{\pi/4} 2\sin^2(\theta)\, d\theta = \int_0^{\pi/4} 2\left(\frac{1-\cos(2\theta)}{2}\right) d\theta$$
$$= \left(\theta - \frac{1}{2}\sin(2\theta)\right)\Big|_0^{\pi/4} = \frac{\pi - 2}{4}.$$

2. (2 points) Evaluate $\iiint_B z^2 e^x \, dV$ where B is the box given by

$$0 \le x \le 1, \quad 1 \le y \le 2, \quad -1 \le z \le 1.$$

Solution: Using Fubini's theorem we evaluate the integral in the order $dxdydz$:

$$\iiint_B z^2 e^x \, dV = \int_{-1}^{1} \int_{1}^{2} \int_{0}^{1} z^2 e^x \, dxdydz$$
$$= \int_{-1}^{1} \int_{1}^{2} z^2 e^x \Big|_{x=0}^{x=1} dydz = \int_{-1}^{1} \int_{1}^{2} z^2(e-1) \, dydz = \int_{-1}^{1} z^2(e-1)y \Big|_{y=1}^{y=2} dz$$
$$= \int_{-1}^{1} z^2(e-1) \, dz = \frac{1}{3} z^3 (e-1) \Big|_{z=-1}^{z=1} = \frac{2}{3}(e-1).$$