MA 213 Worksheet #26

Review for Final!
04/25/19

Chapter 16 Questions: *Taken from Chapter 16 Review, pgs 1148-1149.*

1 (a) Write the definition of the line integral of a scalar function f along a smooth curve C with respect to arc length.

(b) State the Fundamental Theorem for Line Integrals.

(c) What does it mean to say that $\int_C \mathbf{F} \cdot d\mathbf{r}$ is independent of path? If you know that $\int_C \mathbf{F} \cdot d\mathbf{r}$ is independent of path, what can you say about $\mathbf{F}$?

2 Evaluate the line integral: $\int_C yz\cos(x)ds$, where $C: x = t, y = 3\cos(t), z = 3\sin(t), 0 \leq t \leq \pi$

3 Show by example the following is false: If $\mathbf{F}$ and $\mathbf{G}$ are vector fields and div$\mathbf{F}$=div$\mathbf{G}$, then $\mathbf{F} = \mathbf{G}$.

4 Evaluate the line integral: $\int_C xydx + y^2dy + yzdz$, where C is the line segment from $(1, 0, -1)$ to $(3, 4, 2)$.

5 Show that $\mathbf{F}(x, y) = (1 + xy)e^{xy}\mathbf{i} + (e^y + x^2e^{xy})\mathbf{j}$ is a conservative vector field. Then find the function f such that $\mathbf{F} = \nabla f$

6 Use Green's Theorem to evaluate $\int_C x^2ydx - xy^2dy$, where C is the circle $x^2 + y^2 = 4$ with counterclockwise orientation.

7 Use Stokes' Theorem to evaluate $\int_C \mathbf{F} \cdot d\mathbf{r}$, where $\mathbf{F}(x, y, z) = xy\mathbf{i} + yz\mathbf{j} + zx\mathbf{k}$, and C is the triangle with vertices $(1, 0, 0)$, $(0, 1, 0)$, and $(0, 0, 1)$, oriented counter-clockwise as viewed from above.

8 Use the Divergence Theorem to calculate the surface integral $\iint_S \mathbf{F} \cdot \mathbf{S}$, where $\mathbf{F}(x, y, z) = x^3\mathbf{i} + y^3\mathbf{j} + z^3\mathbf{k}$ and S is the surface of the solid bounded by the cylinder $x^2 + y^2 = 1$ and the planes $z = 0$ and $z = 2$.

Chapter 12-15 Questions:

9 *12* - Find an equation of the plane through $(2,1,0)$ and parallel to $x+4y-3z=1$.

10 *12* - Find a vector perpendicular to the plane through the points $A=(1,0,0), B=(2,0,-1), C=(1,4,3)$. Now find the area of triangle ABC.

11 *13.4.17* Find the position vector of a particle that has acceleration vector $\mathbf{a}(t)=2t\mathbf{i}+\sin t\mathbf{j}+\cos 2t\mathbf{k}$, initial velocity $\mathbf{v}(0)=\mathbf{i}$, and initial position $\mathbf{r}(0)=\mathbf{j}$.

12 *14.5.21* - Use the Chain Rule to find $\frac{\partial z}{\partial s}, \frac{\partial z}{\partial t}, \frac{\partial z}{\partial u}$ when $s=4$, $t=2$, and $u=1$.

$$z=x^2+y^2, \quad x=s+2t-u, \quad y=stu^2$$

13 *14.7.31* - Find the absolute maximum and minimum values of f on the set D

$$f(x,y)=x^2+y^2-2x$$

D is the closed triangular region with vertices $(2,0),(0,2)$, and $(0,-2)$.

14 *14.8.5* - The following is an extreme value problem with both a maximum and minimum value. Use Lagrange Multipliers to find the extreme values of the function subject to the given constraint.

$$f(x,y)=xy$$
$$4x^2+y^2=8$$

15 *14.4.19* Given that f is a differentiable function with $f(2,5)=6$, $f_x(2,5)=1$ and $f_y(2,5)=-1$, use a linear approximation to estimate $f(2.2,4.9)$.

16 *15.2.17* Evaluate the integral $\iint_D x\cos y\,dA$, where D is the region bounded by $y=0$, $y=x^2$, and $x=1$.

17 *15.8.27* - Find the volume of the part of the ball $\rho \leq a$ that lies between the cones $\varphi=\pi/6$ and $\varphi=\pi/3$.