Math/Physics 507

Fourier Series, Continued

Extensions: A No-Fault Quiz

Consider now functions with period $2L$. We define

$$f \cdot g = \int_{-L}^{L} f(x)g(x)\,dx$$

and

$$\|f\| = (f \cdot f)^{1/2}$$

in analogy to what we did before.

A plausible set of basis functions is the set

$$\cos(n\pi x/L), n = 0, 1, 2, \cdots$$

$$\sin(n\pi x/L), n = 1, 2, \cdots$$

Thus we look for Fourier series of the form

$$f(x) \sim \frac{a_0}{2} + \sum_{k=1}^{\infty} \left(a_k \cos(k\pi x/L) + b_k \sin(k\pi x/L)\right)$$

1. Check that the functions above are really orthogonal and find the integrals

$$\int_{-L}^{L} \cos^2(n\pi x/L)\,dx$$

$$\int_{-L}^{L} \sin^2(n\pi x/L)\,dx$$

It may be useful to recall the formulas

$$\cos(x)\cos(y) = \frac{1}{2}\left(\cos(x+y) + \cos(x-y)\right)$$

$$\sin(x)\sin(y) = \frac{1}{2}\left(\cos(x-y) - \cos(x+y)\right)$$

$$\sin(x)\cos(y) = \frac{1}{2}\left(\sin(x+y) + \sin(x-y)\right)$$

2. Find formulas for a_k and b_k. Also, if

$$f(x) \sim \frac{a_0}{2} + \sum_{k=1}^{\infty} (a_k \cos(\pi k x/L) + b_k \sin(\pi k x/L))$$

find $\int_{-L}^{L} f(x)^2 \, dx$ in terms of the a_k and b_k.