

I retired from administration and teaching at the end of 2003, but remained active in research. It was a pleasure to be able to attend lengthier meetings and workshops during the academic year instead of turning them down because of other obligations.

The notes from Math 301–305 (Analysis) have been turned into a textbook. This is my second undergraduate analysis textbook. The first, published by Springer in 1973, tried to use the theory of distributions to make everything easier. It was published in Springer’s “Graduate Texts” series and was savaged by an anonymous English reviewer, who read the series title but not the introduction, for not being at graduate level. It has seen little use, even by me.

The new text, *Analysis, an Introduction*, was published by Cambridge in 2004. It is much more classical in approach, and gets to some fun topics including Euler’s product formula for $\sin x$, the Banach–Tarski paradox, and the Heisenberg uncertainty principle. It has been in use for Math 301–305 here but, judging from the yearly sales, sees very little use elsewhere. For anyone who might be interested, the table of contents and a list of corrections are appended.

Per *Ecclesiastes*, “of making many books there is no end.” Through various research projects I was drawn, reluctantly, into a passing acquaintance with special functions (the ones that you don’t learn about in calculus but that have names nonetheless). Some random events led to co-authoring a book on the subject, with Roderick Wong of the City University of Hong Kong, who is an actual expert in the subject. Cambridge University Press took it on as well, and it is now available: *Special Functions, a Graduate Text*. The table of contents is listed at the end below, followed by some comments and some minor corrections.

Analysis, an Introduction – Contents:

1. Introduction
2. The Real and Complex Numbers
3. Real and Complex Sequences
4. Series
5. Power Series
6. Metric Spaces
7. Continuous Functions
8. Calculus
9. Some Special Functions
10. Lebesgue Measure on the Line
11. Lebesgue Integration on the Line
12. Function Spaces
13. Fourier Series
14. Applications of Fourier Series
15. Ordinary Differential Equations

I am indebted to Mary Pugh, Gerard Misiolek, and José Rodrigo for most of the following corrections. And I am especially indebted to Eric Belsley, whose detailed comments on the notes for chapters 1 – 9 are the chief reason there are so few corrections for those chapters. (The exercises on page 130 were added later!)

Corrections to “Analysis, an Introduction”

p. 6, eqn. (14): $(m-n)+(p-q)=(m+p)-(n+q)$

p. 7, line 9: ...and satisfies M1, M2,

p. 26, exercise 2: (ii) $1, x, x^2, \dots, x^n$ are linearly dependent over \mathbb{Q} .

exercise 2: (iv) there are real numbers y_1, y_2, \dots, y_n such that ...

p. 28, line 16: $= |z|^2 + 2\operatorname{Re}(z\bar{w}) + |w|^2 \leq \dots$

p. 66, exercise 4: right side should be $1/(1-z)^{k+1}$.

exercise 5: Under the given assumptions, show there is a power series $\sum_{n=0}^{\infty} b_n z^n$ with radius of convergence at least $R-r$ such that

$$\sum_{n=0}^{\infty} a_n z^n = \sum_{n=0}^{\infty} b_n (z - z_0)^n, \quad |z - z_0| < R - r.$$

p. 103, line 3: $= f'(c)[g(b) - g(a)] - g'(c)[f(b) - f(a)];$

line 4: divide by $[g(b) - g(a)]g'(c)$.

p. 115, lines 1-2: Suppose that f is a real-valued function ...

p. 124, equation (14): $z^n + a_{n-1}z^{n-1} + \dots$

p. 130, exercise 8: $F(a, b, c; z)$

exercise 9: $F(a, b, b; z)$

exercise 10: $\int_0^1 t^{b-1}(1-t)^{c-b-1}(1-tz)^{-a} dt$

p. 137, line 6: $\dots + m^*((E \cap (A \cup B)) \cap A^c)$

p. 149, line 8: ... for $x \in [0, 1] \cap C^c$...

p. 152, line 13: $\int \int (f + g)$ should be $\int (f + g)$

p. 170, line 9: $\{h^* > \delta/2\}, \quad \{h > \delta/2\}$

p. 171, line 5: ... and define a subfamily \mathcal{I}_{n+1} ...

line 6: ... if \mathcal{I}_{n+1} is empty ...

line 11: ... implies that $|I_k| \geq |I|/2$...

line -10: ... $I \subset I_n^*$ for some n.

p. 178, equation (17): omit $1/2\pi$

p. 181, line -9: $\sum_{n=0}^{2N} (e^{ix})^n$

line -11: ... as long as $e^{ix} \neq 1$,

- p. 183, line 9: $S_N f(x_0) - f(x_0) = \dots$
line 10: $\dots [f(x_0 - y) - f(x_0)] dy$
equation (31): $\sin\left(\left[N + \frac{1}{2}\right] y\right)$
- p. 186, line -5,-4: \dots property (ii) in Proposition 13.9 \dots
line -2: $\int_{|y| < \delta}$
line -1: $\int_{\delta < |y| < \pi}$
- p. 189, line -4: See Exercise 9 of Section 12D.
- p. 192, line 8: $f_N(x) = \sum_{-N}^N a_n e^{inx}$
- p. 192, line -2: $a_0 = 0$
- p. 193, line 3: Use Exercise 5 of this section and \dots
- p. 194, exercise 10: $f(x_0-) = \lim_{x \rightarrow x_0, x < x_0} f(x)$; \dots
- p. 196, exercise 4: $g_n(x) = \dots$
- p. 201, equation (6): $0 \leq t \leq 2\pi$
- p. 211, line 5: \dots let $I_L = [-\frac{1}{2}L, \frac{1}{2}L]$ \dots
line 8: $\int_{-L/2}^{L/2}$
- p. 214, exercise 6: $f(x) = \frac{1}{\sqrt{2\pi}} e^{-x^2/2}$
- p. 215, exercise 9: \dots (assume (46) and use Dominated Convergence).
- p. 217, equation (57): $V_T = \|(T - E_T I)\psi\|^2$
- p. 227, line 11: \dots but it may not be unique.
- p. 234, line 1: $\mathbf{f}(x, \mathbf{y})$
- p. 238, proof of Lemma 238. It might seem at first that the sets removed from \tilde{A}_2 and \tilde{A}_3 to form A_2 and A_4 are empty, but there is a nonzero vector that is fixed by σ .
- p. 239, statement of Theorem: there should be four B'_j and three C'_j : drop C'_4 and take

$$B'_j = B_j \cap (D_f \cup D_\infty), \quad j = 1, 2; \quad B'_j = B_{j-2} \cup D_g, \quad j = 3, 4.$$

Then the first four A'_j correspond to the B'_j and the last three to the C'_j .

Also: $f : D_\infty \rightarrow A_\infty$ and $g : A_\infty \rightarrow D_\infty$ are bijective (but not inverses of each other).

p. 243, Section 3A: Omit 12, and renumber 13 – 17 as 12 – 16.

p. 251, Section 13B, 6 (b): \dots the interval $[b, b + 2\pi)$ \dots

p. 252, Section 13G: 1, 2. Use Theorems 13.12 and 13.14 and Proposition 13.4. Note that a continuous periodic function is an L^2 -periodic function. For exercise 1 (b), continuity or lack of continuity can be seen by comparison with a multiple of the square wave function in §14A.

Special Functions, a Graduate Text – Contents:

1. Introduction
 2. Gamma, beta, zeta
 3. Second–order differential equations
 4. Orthogonal polynomials
 5. Discrete orthogonal polynomials
 6. Confluent hypergeometric functions
 7. Cylinder functions
 8. Hypergeometric functions
 9. Spherical functions
 10. Asymptotics
 11. Elliptic functions
- Appendix A. Complex analysis
Appendix B. Fourier analysis

Comments: Meijer G –functions

At the time this book was written I knew nothing about these functions. They are certain normalized solutions to the generalized hypergeometric equation, which, as a family, have very useful closure properties. They are particularly important in computing various indefinite integrals. Pending a (putative) second edition of the book, see the following for basic information and references:

R. Beals and J. Szmigielski, *Meijer G –functions, a gentle introduction*, Notices of the AMS **60** (2013), 866-872.

Corrections to “Special Functions”

p. 5, line 13 ... functions of (a, c)

p. 60, (3.1.11) missing radical: $p_1'(y)/\sqrt{2p_1(y)}$

p.267, lines 3, 6, 7 second comma should be a semicolon