Math 641

Homework 3

Due Monday, September 22

A note on notation: The differential of a mapping f at a point q in its domain is sometimes denoted $f'(q)$ and also sometimes df_q.

1. Let M be a regular surface in $\mathbb{R}^3$. Show that for any $p \in M$ there is a neighborhood V of p in M so that V is the graph of a differentiable function that has one of the following three forms: $z = f(x, y)$, $y = g(x, z)$, or $x = h(y, z)$. *Hint*: there is a coordinate map $\mathbf{x} : U \subset \mathbb{R}^2 \to \mathbb{R}^3$ with $\mathbf{x}(0) = p$ and $d\mathbf{x}_0$ has maximal rank.

2. Suppose that M is a differentiable manifold in the sense of do Carmo, Definition 2.1 page 2. A subset A of M is *open* if for every α for which $A \cap \mathbf{x}_\alpha(U_\alpha)$ is nonempty, the set $\mathbf{x}_\alpha^{-1}(A \cap \mathbf{x}_\alpha(U_\alpha))$ is an open subset of $\mathbb{R}^n$. Show that the collection of such open sets does indeed form a topology on M, i.e.:

   (a) The sets M and $\emptyset$ are open

   (b) An arbitrary union of open sets in M is open

   (c) A finite intersection of open sets in M is open

3. do Carmo, page 32, problem 1, part (a)