Math 641
Homework 5
Due Friday, October 16

1. This problem concerns the matrix group $SL_n(\mathbb{R})$, defined as the invertible $n \times n$ matrices with determinant 1. Recall that $GL_n(\mathbb{R})$ is the group of invertible $n \times n$ matrices, and may be viewed as an open subset of $\mathbb{R}^n \times \ldots \times \mathbb{R}^n$ (n-fold Cartesian product) by identifying a matrix A with rows $a_i \in \mathbb{R}^n$, $1 \leq i \leq n$, with the point $(a_1, \ldots, a_n) \in \mathbb{R}^n \times \ldots \times \mathbb{R}^n$. Thus $GL_n(\mathbb{R})$ is a smooth manifold of dimension n^2.

   (a) Show that the determinant function

   $$\det : \mathbb{R}^n \times \ldots \times \mathbb{R}^n \to \mathbb{R}$$
   $$(a_1, \ldots, a_n) \mapsto \det \begin{bmatrix} a_1 \\ \vdots \\ a_n \end{bmatrix}$$

   has differential

   $$D\,(\det)\,(a_1, \ldots, a_n)(x_1, \ldots, x_n) = \sum_{i=1}^{n} \det \begin{bmatrix} a_1 \\ \vdots \\ x_i \\ \vdots \\ x_n \end{bmatrix}.$$

   (b) The matrix group $SL_n(\mathbb{R})$ is the set of all $A \in GL_n(\mathbb{R})$ with $\det(A) = 1$. Using the result of part (a), show that $SL_n(\mathbb{R})$ is a smooth submanifold of $GL_n(\mathbb{R})$ of dimension $n^2 - 1$.

2. do Carmo, page 45, problem 1

3. do Carmo, page 46, problem 4