MA 114 Worksheet #19: Volumes II

1. (a) Write a general integral to compute the volume of a solid obtained by rotating the region under $y = f(x)$ over the interval $[a, b]$ about the y-axis using the method of cylindrical shells.

   (b) If you use the disk method to compute the same volume, are you integrating with respect to x or y? Why?

2. Sketch the enclosed region and use the Shell Method to calculate the volume of rotation about the y-axis.

   (a) $y = 3x - 2$, $y = 6 - x$, $x = 0$

   (b) $y = x^2$, $y = 8 - x^2$, $x = 0$, for $x \geq 0$

   (c) $y = 8 - x^3$, $y = 8 - 4x$, for $x \geq 0$

3. For each of the following, use disks or washers to find the an integral expression for the volume of the region. Evaluate the integrals for parts (a) and (d).

   (a) R is region bounded by $y = 1 - x^2$ and $y = 0$; about the x-axis.

   (b) R is region bounded by $y = \dfrac{1}{x}$, $x = 1$, $x = 2$, and $y = 0$; about the x-axis.

   (c) R is region bounded by $x = 2\sqrt{y}$, $x = 0$, and $y = 9$; about the y-axis.

   (d) R is region bounded by $y = 1 - x^2$ and $y = 0$; about the line $y = -1$.

   (e) Between the regions in part (a) and part (d), which volume is bigger? Why?

   (f) R is region bounded by $y = e^{-x}$, $y = 1$, and $x = 2$; about the line $y = 2$.

   (g) R is region bounded by $y = x$ and $y = \sqrt{x}$; about the line $x = 2$.

4. A soda glass has the shape of the surface generated by revolving the graph of $y = 6x^2$ for $0 \leq x \leq 1$ about the y-axis. Soda is extracted from the glass through a straw at the rate of $1/2$ cubic inch per second. How fast is the soda level in the glass dropping when the level is 2 inches? (Answer should be implicitly in units of inches per second.)

5. The torus is the solid obtained by rotating the circle $(x-a)^2 + y^2 = b^2$ around the y-axis (assume that $a > b$). Show that it has volume $2\pi^2 ab^2$.
   [Hint: Draw a picture, set up the problem and evaluate the integral by interpreting it as the area of a circle.]