Quiz 2

Name: _____ Section and/or TA: _____

Answer all questions in a clear and concise manner. Unsupported answers will receive *no credit*.

1. Evaluate the following integrals and show your steps.

a) (2 points) $\int \sin^3 x \cos^2 x \, dx$

Solution: Let $u = \cos x$. Then $du = -\sin x \, dx$ so

$$\begin{aligned}\int \sin^3 x \cos^2 x \, dx &= \int (1 - \cos^2 x) \cos^2 x \, \sin x \, dx \\ &= -\int (1 - u^2) u^2 \, du = \int (u^4 - u^2) \, dx \\ &= \frac{1}{5}u^5 - \frac{1}{3}u^3 + C = \frac{1}{5}\cos^5 x - \frac{1}{3}\cos^3 x + C\end{aligned}$$

b) (2 points) $\int \dfrac{4 \, dx}{(x^2+4)^{\frac{3}{2}}}$

Solution:

Let $x = 2\tan\theta$, where $-\pi/2 < \theta < \pi/2$. Then $dx = 2\sec^2\theta \, d\theta$ and

$$\sqrt{x^2+4} = \sqrt{4\tan^2\theta + 4} = \sqrt{4(1+\tan^2\theta)} = \sqrt{4\sec^2\theta} = 2\sec\theta$$

since $\sec\theta > 0$. Hence

$$\begin{aligned}\int \frac{4 \, dx}{(x^2+4)^{\frac{3}{2}}} &= \int \frac{8\sec^2\theta}{(2\sec\theta)^3} \, d\theta = \int \frac{8\sec^2\theta}{8\sec^3\theta} \, d\theta = \int \frac{1}{\sec\theta} \, d\theta \\ &= \int \cos\theta \, d\theta = \sin\theta + C = \frac{x}{\sqrt{x^2+4}} + C.\end{aligned}$$

The last equation is obtained by drawing a triangle with sides 2, x and $\sqrt{x^2+4}$.