

Quiz 3

Name: _____ Section and/or TA: _____

Answer all questions in a clear and concise manner. Unsupported answers will receive *no credit*.

1. (2 points) Compute the integral

$$\int \frac{4x}{(x+2)(x^2+4)} dx.$$

Solution: We write the partial fraction decomposition

$$\frac{4x}{(x+2)(x^2+4)} = \frac{A}{x+2} + \frac{Bx+C}{x^2+4}$$

which becomes

$$4x = A(x^2+4) + (Bx+C)(x+2)$$

Letting $x = -2$ implies that $A = -1$, so $B = 1$ and $C = 2$. We thus obtain

$$\int \frac{4x}{(x+2)(x^2+4)} dx = -\ln|x+2| + \frac{1}{2} \ln(x^2+4) + \arctan \frac{x}{2}$$

2. (2 points) Use Simpson's rule (with $n = 4$) to approximate $\int_0^2 \sqrt{4-x^2} dx$. Calculate the error in this estimate by finding the exact value of this integral. (Hint: sketch $\sqrt{4-x^2}$).

Solution:

$$\begin{aligned} \int_0^2 \sqrt{4-x^2} dx &\approx \frac{2-0}{4 \cdot 3} [\sqrt{4-0} + 4\sqrt{4-0.25} + 2\sqrt{4-1} + 4\sqrt{4-1.25} + \sqrt{4-4}] \\ &= \frac{1}{6} [2 + 4\sqrt{3.75} + 2\sqrt{3} + 4\sqrt{2.75} + 0] \approx 3.307 \\ \int_0^2 \sqrt{4-x^2} dx &= \pi \Rightarrow \text{error} \approx .1656 \end{aligned}$$