C

1. The slope of this line

$$6x - 2y = 7$$

is

(a) negative
(b) undefined
(c) positive
(d) zero

D

2. Your band just recorded its demo. The cost to record and make burn x copies of the CD is $C(x) = 2400 + 2x$. You expect to sell the CD's for \$8 each. Determine the fixed cost.

(a) \$400
(b) \$8
(c) \$6
(d) \$2
(e) \$2400

Use this information for the next two questions.
For a given product, the supply curve is given by

$$5x - p + 300 = 0$$

and the demand curve by

$$3x + p - 1200 = 0$$

where p is the price in dollars, and x is the quantity.

E

3. You want to determine how many items consumers are willing to buy if the price were \$400. To determine this, you would

(a) Determine the value of x at which the supply curve and the demand curve intersect.
(b) Set $x = 400$ in $3x + p - 1200 = 0$ and solve for p.
(c) Set $x = 400$ in $5x - p + 300 = 0$ and solve for p.
(d) Set $p = 400$ in $5x - p + 300 = 0$ and solve for x.
(e) Set $p = 400$ in $3x + p - 1200 = 0$ and solve for x.

D

4. According to the above supply and demand curves, at what price will producers refuse to manufacture?

(a) \$1200. Put $x = 0$ in $3x + p - 1200 = 0$ and solve for p.
(b) \$862.50. Determine the value of p at which the supply curve and the demand curve intersect.
(c) \$400. Put $p = 0$ in $5x - p + 300 = 0$ and solve for x.
(d) \$300. Put $x = 0$ in $5x - p + 300 = 0$ and solve for p.
(e) \$60. Put $p = 0$ in $5x - p + 300 = 0$ and solve for x.

D

5. Is this matrix in row reduced form?

$$\left[\begin{array}{ccc|c} 1 & 2 & 0 & 0 \\ 0 & 0 & 1 & 3 \\ 0 & 0 & 2 & 1 \end{array}\right]$$

(a) No, because the entry in the first row, third column is equal to 0

(b) No, because the entry in the second row, second column is equal to 0.

(c) Yes, this matrix is already in row reduced form.

(d) No, because there is a nonzero entry directly below the pivot in row 2, column 3.

(e) No, because the entry in the first row on the right side of the vertical bar is equal to 0.

D

6. Consider the system of equations

$$\begin{array}{rcrcrcl} x & + & 2y & - & 3z & = & 2 \\ 3x & & & & 2z & = & 4 \end{array}$$

You use the row operation $R_2 \mapsto R_2 - 3R_1$. What is the new second row?

(a) $-6y + 7z = -2$

(b) $7z = -2$

(c) $6y - 7z = -2$

(d) $-6y + 11z = -2$

(e) $6y + 7z = 10$

B

7. The second matrix is the result of applying two elementary row operations to the first matrix.

$$\left[\begin{array}{ccc|c} 1 & 0 & 9 & 12 \\ -2 & 2 & 1 & 3 \\ 1 & 2 & -3 & 8 \end{array}\right] \Longrightarrow \left[\begin{array}{ccc|c} 1 & 0 & 9 & 12 \\ 0 & 2 & 19 & 27 \\ 0 & 2 & -12 & -4 \end{array}\right]$$

What are the row operations?

(a) $R_1 \mapsto R_2 + 2R_1$ and $R_3 \mapsto R_3 + R_1$

(b) $R_2 \mapsto R_2 + 2R_1$ and $R_3 \mapsto R_3 - R_1$

(c) $R_2 \mapsto 2R_2 + R_1$ and $R_3 \mapsto R_3 + R_1$

(d) $R_2 \mapsto R_2 - 2R_1$ and $R_3 \mapsto R_3 - R_1$

(e) $R_2 \mapsto R_2 - 2R_1$ and $R_3 \mapsto R_3 + R_1$

8. The following augmented matrix arises while you are solving a system of equations in the variables x, y, and z.

$$\left[\begin{array}{ccc|c} 1 & 1 & 6 & 12 \\ 0 & 0 & 5 & 3 \\ 0 & 0 & 0 & 8 \end{array}\right]$$

Choose the most correct statement.

(a) System has no solution since the first nonzero entry in row 2 is not equal to 1.

(b) System has no solution since bottom row says $0x + 0y + 0z = 8$.

(c) System has no solution since there is no pivot in the second column.

(d) We need to do more row operations before we can determine if there is no solution.

(e) System has infinitely many solutions since the bottom row says $0x + 0y + 0z = 8$.

9. C

$$\begin{array}{rcrcl} 2x & + & y & = & 6 \\ 4x & + & ky & = & 15 \end{array}$$

Determine the value of k for which the system has either no solution or infinitely many solutions.

(a) $k = -2$ and system has infinitely many solutions.

(b) $k = 2$ and system has infinitely many solutions.

(c) $k = 2$ and system has no solution.

(d) $k = 0$ and system has no solution.

(e) $k = -2$ and system has no solution.

A 10. A matrix A has n rows and m columns. Matrix B has p rows and q columns. Which statement conerning the product AB is true?

(a) The product exists only if $m = p$. In this case, AB has n rows and q columns.

(b) The product exists only if $n = p$. In this case, AB has m rows and q columns.

(c) The product exists only if $m = p$. In this case, AB has m rows and p columns.

(d) The product exists only if $n = q$. In this case, AB has m rows and p columns.

(e) The product exists only if $m = n$. In this case, AB has p rows and q columns.

C 11. You wish to solve the system of equations

$$\begin{array}{rcrcrcrcl} x & + & 2y & + & z & + & w & = & 5 \\ x & + & 2y & - & z & + & 3w & = & 1 \end{array}$$

You ask a computer to solve this problem, and it gives you the following augmented matrix

$$\left[\begin{array}{cccc|c} 1 & 2 & 0 & 2 & 3 \\ 0 & 0 & 1 & -1 & 2 \end{array}\right]$$

Finish solving the problem.

(a) $(x, y, z, w) = (3 - 2s - 2t, 2 + t, s, t)$

(b) $(x, y, z, w) = (3 + 2s + 2t, s, 2 - t, t)$

(c) $(x, y, z, w) = (3 - 2s - 2t, s, 2 + t, t)$

(d) $(x, y, z, w) = (s, t, 3 - 2s - 2t, 2 + t)$

(e) $(x, y, z, w) = (s, t, 3 + 2s + 2t, 2 - t)$

B 12. Determine the value of u, given that

$$\begin{bmatrix} x & -2 \\ 3 & y \end{bmatrix} + \begin{bmatrix} -2 & z \\ 1 & 2 \end{bmatrix} = 2 \cdot \begin{bmatrix} 2 & -1 \\ u & 2 \end{bmatrix}$$

(a) 1 (b) 2 (c) 3 (d) 4 (e) 5

13. (8 points) **Set up, but do not solve this problem. To receive credit, your variables must be clearly labelled.** A rent-a-car company has \$2.1 million dollars to buy a fleet of new cars. They can purchase compact cars, mid-sized cars, and full-sized cars. Each compact car costs \$12,000, each mid-sized car costs \$18,000, and each full-sized car costs \$33,000. The company wants to purchase 100 cars. Also, the total number of mid-sized and full-sized cars, combined, should be the same as the number of compact cars. Write down a system of equations which will help the company's manager decide how many cars of each type to order.

Let C = # compact cars, M = # midsized cars, F = # full sized cars

Then
$$\begin{cases} C + M + F = 100 & \text{(Total \# cars)} \\ 12000C + 18000M + 33000F = 2{,}100{,}000 & \text{(Total Budget)} \\ C = M + F \end{cases}$$

14. (8 points) Determine the product: $\begin{bmatrix} 2 & 1 & 2 \\ 3 & 2 & 4 \end{bmatrix} \begin{bmatrix} -1 & 2 \\ 4 & 3 \\ 0 & 1 \end{bmatrix}$

2×3 $\quad$ 3×2 $\quad$ Product will be 2×2.

Show work to receive credit!

$$\begin{bmatrix} 2\cdot(-1)+1\cdot4+2\cdot0 & 2\cdot2+1\cdot3+2\cdot1 \\ 3(-1)+2\cdot4+4\cdot0 & 3\cdot2+2\cdot6+4\cdot1 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & 9 \\ 5 & 16 \end{bmatrix}$$

15. (12 points) The numbers of checking accounts and savings accounts at the Main Office, the Westside Branch, and the Eastside Branch of the Central Bank are recorded in this matrix.

$$A = \begin{bmatrix} 2000 & 1500 \\ 1200 & 1100 \\ 750 & 950 \end{bmatrix}$$

Over the course of the year, at the Main Office 120 new checking accounts were opened, 60 checking accounts were closed, 100 new savings were opened and 20 savings accounts were closed. At the Westside branch, 40 new checking accounts were opened, 20 checking accounts were ~~opened~~ closed, 75 new savings accounts were opened, and 30 savings accounts were closed. At the Eastside Branch, 80 new checking accounts were opened, 30 checking accounts were closed, 100 new savings accounts were opened, and 25 savings accounts were closed.

(i.) Write down a matrix B which represents the number of newly opened accounts.

$$B = \begin{bmatrix} 120 & 100 \\ 40 & 75 \\ 80 & 100 \end{bmatrix}$$

(ii.) Write down a matrix C which represents the number of recently closed accounts.

$$C = \begin{bmatrix} 60 & 20 \\ 20 & 30 \\ 30 & 25 \end{bmatrix}$$

(iii.) Matrix D represents the number of accounts at each branch at the end of the year. Write down an expression for D in terms of A, B, C.

Current + New − closed.

$$D = A + B - C$$

(iv.) Compute and simplify D.

$$\begin{bmatrix} 2000 & 1500 \\ 1200 & 1100 \\ 750 & 950 \end{bmatrix} + \begin{bmatrix} 120 & 100 \\ 40 & 75 \\ 80 & 100 \end{bmatrix} - \begin{bmatrix} 60 & 20 \\ 20 & 30 \\ 30 & 25 \end{bmatrix}$$

$$= \begin{bmatrix} 2060 & 1580 \\ 1220 & 1145 \\ 800 & 1025 \end{bmatrix}$$

16. (12 points) Use the Gauss-Jordan method to determine the inverse of this matrix. **Show work to receive credit! Denote row operations using $R_2 \mapsto R_2 + 5 \cdot R_1$ notation.**

$$\begin{bmatrix} 1 & -2 & 1 \\ 2 & -3 & -4 \\ 0 & 0 & -1 \end{bmatrix}$$

$\left[\begin{array}{ccc|ccc} 1 & -2 & 1 & 1 & 0 & 0 \\ 2 & -3 & -4 & 0 & 1 & 0 \\ 0 & 0 & -1 & 0 & 0 & 1 \end{array}\right] \begin{array}{l} R_2 \mapsto R_2 - 2R_1 \\ \longrightarrow \\ R_3 \mapsto -R_3 \end{array} \left[\begin{array}{ccc|ccc} 1 & -2 & 1 & 1 & 0 & 0 \\ 0 & 1 & -6 & -2 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & -1 \end{array}\right]$

$R_1 \mapsto R_1 - R_3$
$R_2 \mapsto R_2 + 6R_3$

$\left[\begin{array}{ccc|ccc} 1 & -2 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & -2 & 1 & -6 \\ 0 & 0 & 1 & 0 & 0 & -1 \end{array}\right] \begin{array}{l} R_1 \mapsto R_1 + 2R_2 \\ \longrightarrow \end{array}$

$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -3 & 2 & -11 \\ 0 & 1 & 0 & -2 & 1 & -6 \\ 0 & 0 & 1 & 0 & 0 & -1 \end{array}\right]$

So $A^{-1} = \begin{bmatrix} -3 & 2 & -11 \\ -2 & 1 & -6 \\ 0 & 0 & -1 \end{bmatrix}$

17. (12 points) Your Rent-A-Car company has \$1,512,000 to purchase 60 new cars. You can choose between compact, mid-sized, and full-sized cars. The prices of the cars are \$18,000, \$28,800, and \$39,600 respectively. You let x denote the number of compact cars, y the number of mid-sized, and z the number of full-sized cars. You ask your assistant to determine the number of each type of car. Your assistant applies the Gauss-Jordan method and sends you the results of the computation:

$$\begin{aligned} x &= 20 + t \\ y &= 40 - 2t \\ z &= t \end{aligned}$$

(i.) Determine one possible option. Write your answer in plain english!

We can get a specific option by choosing a value of t. One option is to say $t = 0$, in which case $x = 20$, $y = 40$, $z = 0$. In other words, 20 compact cars, 40 midsize & 0 Full size.

(ii.) Find the option with the greatest number of compact cars. (Hint: what restrictions must be placed on the free parameter t?)

We want $20+t$ as large as possible, but we need x, y, z all to remain ≥ 0. Now, $y = 0$ $\Rightarrow 40 - 2t \Rightarrow t = 20$. This results in 40 compact cars, 0 mid-size, and 20 full-size