

# Exam 2

MA 162: Finite Mathematics  
University of Kentucky

October 28, 2013

**Directions:**

- Do not remove this page—you will turn in the entire exam.
- Complete this exam using only a pen or pencil and a simple calculator (not a cellphone).
- The point value for each question is shown in the exam booklet.
- On free response questions you must show all work in order to receive credit. Unjustified answers will receive no credit!
- If asked to explain, you must write clearly and in complete sentences.
- Use the back side of the exam for scrap work.

Printed Name: Solutions Section: \_\_\_\_\_

**Do Not Write Anything Here**

Question	Points Possible	Score
Page 1	15 points	
Page 2	15 points	
Page 3	10 points	
Page 4	20 points	
Page 5	20 points	
Page 6	20 points	
Total	100 points	

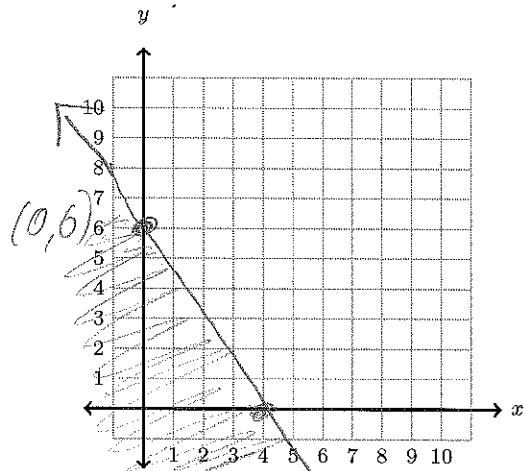


1 (5 points) Find the graphical solution of this inequality:

$$6x + 4y \leq 24$$

(Clearly label intercepts on the graph)

Test point:  $(0,0)$   
 $6 \cdot 0 + 4 \cdot 0 \leq 24 \checkmark$



2 (5 points)

Shade the region defined by this system of inequalities:

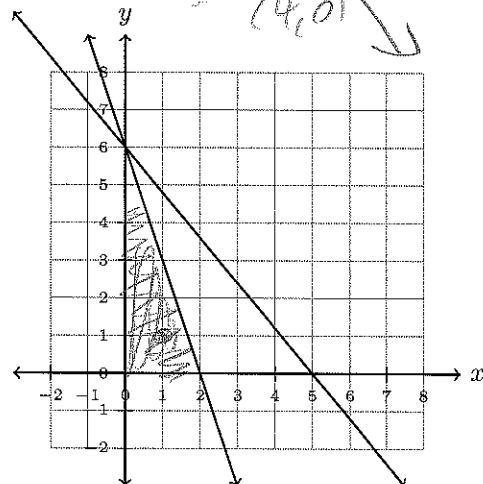
$$6x + 5y \leq 30$$

$$3x + y \leq 6$$

$$x \geq 0$$

$$y \geq 0$$

Test points  
 $(1,1)$ :  
 $6 \cdot 1 + 5 \cdot 1 \leq 30 \checkmark$   
 $3 \cdot 1 + 1 \leq 6 \checkmark$   
 $1 \geq 0 \checkmark$   
 $1 \geq 0 \checkmark$



$$3x + y = 6$$

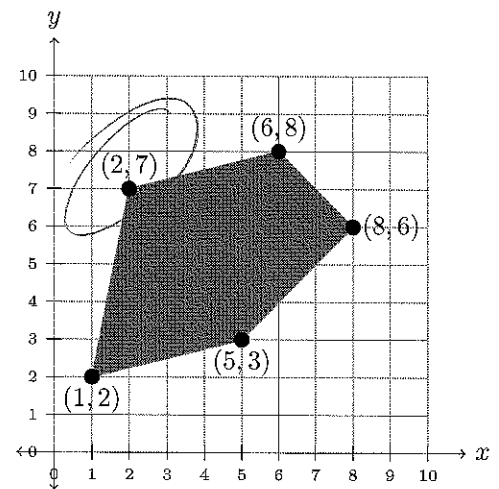
$$6x + 5y = 30$$

3 (5 points)

Determine the minimum value of  $P = -x + 2y$  on the sketched region.

x	y	P
1	2	3
4	3	1
8	6	4
6	8	10
2	7	12

Max Value  
is 12



4 (10 points) Read the whole problem before you start computing. You are NOT asked to solve this problem in its entirety!

ACME Packers has up to \$100 million to invest in three building projects.

- A new packaging plant
- A new processing plant
- A new employee workout center

A new packaging plant is expected to return 10% on its investment, a new processing plant is expected to return 17.5% on its investment, while a new workout center is expected to return 15% on its investment.

In order not to neglect part of the company, ACME Packers decides to invest no more than 20% of the total investment in the workout center. They also decide that the investment in the processing plant and the workout center should be at most 60% of the investment in the packaging plant.

Help ACME Packers determine how to maximize the total return on their investment. A complete solution should

- Define the relevant variables;
- Identify all of the constraints, in mathematical form;
- Identify the objective.

You do NOT need to find the optimal solution—ACME packers will ask one of their interns to do that part.

*Let*  $x =$  Amt invested in packaging plant  
 $y =$  Amt in processing plant  
 $z =$  Amt in workout center.

Constraints:  $x, y, z \geq 0,$   $x + y + z \leq 100,000,000$   
 $z \leq 0.20(x + y + z)$   
 $y + z \leq 0.60(x + y + z)$

Objective: Maximize:  $0.10x + .175y + .20z$ .

5 (5 points) Consider the following simplex tableau.

$x$	$y$	$z$	$u$	$v$	$P$	Constant
0	4	1	3	-7	0	3
1	2	0	-1	2	0	4
0	0	0	2	5	1	10

Which variables are basic variables?

Basic:  $x, z, P$

Which variables are non-basic variables?

Non-Basic:  $y, u, v$



6 (5 points) The following simplex tableau is not in final form. Identify the next pivot to be used in the simplex method. Explain why you chose that column. Explain why you chose that row.

$x$	$y$	$u$	$v$	$P$	Constant
3	5	1	0	0	60
7	2	0	1	0	42
-9	-4	0	0	1	11

$$60/3 = 20$$

$$42/7 = 6$$

Column  $x$  since most negative entry in bottom row is in column  $x$

Row 2 since ratio  $42/7 < 60/3$

7 (5 points) Let  $U$  denote the set of all students in the business college of a certain university. Let

- $A = \{x \text{ is in } U \mid x \text{ had taken a course in accounting}\}$
- $E = \{x \text{ is in } U \mid x \text{ had taken a course in economics}\}$
- $M = \{x \text{ is in } U \mid x \text{ had taken a course in marketing}\}$

Which statement best describes the set  $(A \cap E^c) \cup M$ ?

- (A) The set of students who have taken marketing, and either took accounting or did not take economics.
- (B) The set of students who have either taken accounting and not economics, or who have taken marketing.
- (C) The set of students who have taken accounting and marketing, but have not taken economics.
- (D) The set of students who have taken neither accounting nor economics, but have taken marketing.
- (E) The set of students who have either taken marketing or have not taken accounting and economics.

$A \cap E^c \Rightarrow \text{Acc AND not Econ}$

It's between B & C.  
But  $(A \cap E^c) \cup M \Rightarrow (\text{Acc} \cap \text{Not Econ}) \cup \text{Marketing}$



8 (10 points)  $A = \{2, 4, 6, 8\}$ ,  $B = \{3, 4, 5, 6, 7\}$ ,  $C = \{1, 2, 3, 4, 5, 6\}$

(a) Determine  $B \cap C$

$$= \{3, 4, 5, 6\}$$

(b) Determine  $A \cup B$

$$\{2, 3, 4, 5, 6, 7, 8\}$$

(c) Determine  $A - B$

$$\{2, 8\}$$

(d) Determine  $A \cup (B \cap C)$

$$\{2, 3, 4, 5, 6, 8\}$$

9 (5 points) In a survey of 120 consumers conducted in a shopping mall, 80 consumers indicated that they buy brand A of a certain product, 68 consumers indicated that they buy brand B of that product, and 42 indicate that they buy both brands. How many consumers from this survey purchase brand A only?

$$n(A) = 80$$

$$n(B) = 68$$

$$n(A \cap B) = 42$$

$$\text{So } n(A \cup B) = 80 + 68 - 42 = 106$$

$$A \text{ only} \Rightarrow (A \cup B) - B \Rightarrow 106 - 68 = 38$$



10 (5 points) A survey of 100 college students who frequent the reading lounge of a university revealed the following:

- 40 read *Time*
- 30 read *Newsweek*
- 25 read *The Economist*
- 15 read *Time* and *Newsweek*
- 12 read *Time* and *The Economist*
- 10 read *Newsweek* and *The Economist*
- 4 read all three magazines.

Determine how many of the 100 students read at least one of these magazines.

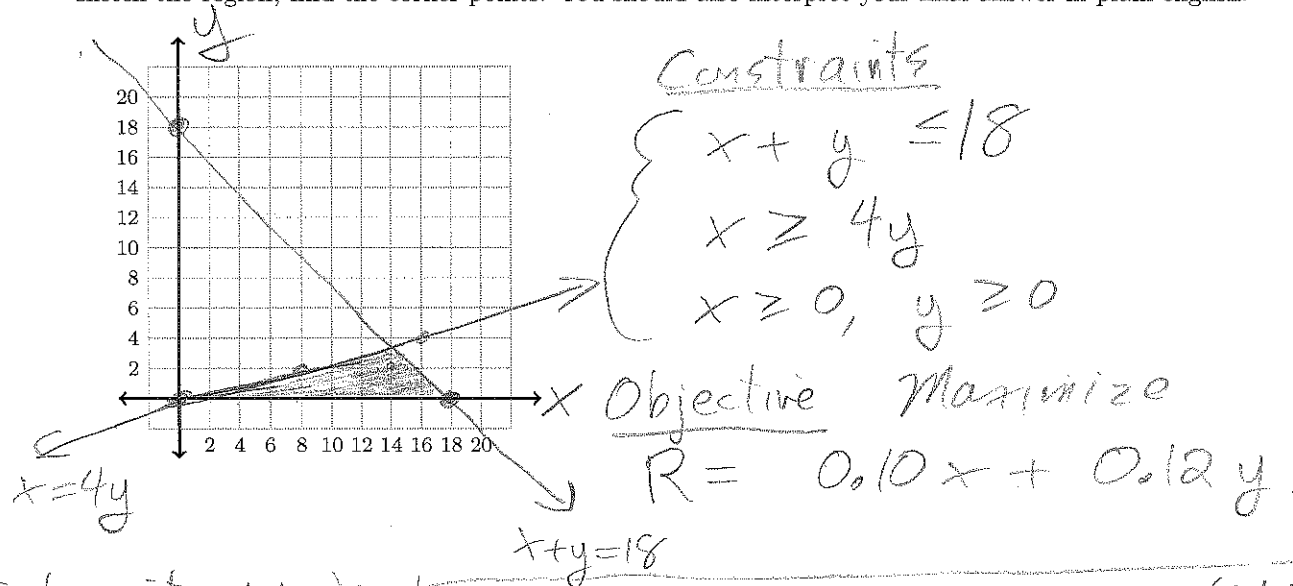
$$\begin{aligned} n(N \cup T \cup E) &= n(N) + n(T) + n(E) \\ &\quad - n(N \cap T) - n(N \cap E) - n(T \cap E) \\ &\quad + n(N \cap T \cap E) \\ &= 40 + 30 + 25 - 15 - 12 - 10 + 4 = 62 \end{aligned}$$



11 (20 points) Use the method of corners to solve this problem

- Madison Finance has a total of \$18 million earmarked for homeowner loans and auto loans, where  $x$  is homeowner loans in millions of dollars and  $y$  is auto loans in millions of dollars.
- On the average, homeowner loans have a 10% annual rate of return, whereas auto loans yield a 12% annual rate of return.
- Management has also stipulated that the total amount of homeowner loans should be greater than or equal to 4 times the total amount of automobile loans.
- Determine the total amount of loans of each type Madison should extend to each category to maximize its returns  $P$  in millions of dollars.

To receive full credit, you must show all work. You will need to express the required inequalities and equalities, sketch the region, find the corner points. You should also interpret your final answer in plain english.



Test point: (14, 2):  
 $14 + 2 \leq 18 \checkmark$   
 $14 \geq 4 \cdot 2 \checkmark$   
 $14 \geq 0, \checkmark 2 \geq 0 \checkmark$

Corners: (0, 0), (18, 0), (14.4, 3.6)

$$\begin{cases} x + y = 18 \\ x = 4y \end{cases} \Rightarrow \begin{cases} 4y + y = 18 \\ x = 4y \end{cases} \Rightarrow \begin{cases} y = 18/5 = 3.6 \\ x = 4 \cdot 18/5 = 14.4 \end{cases}$$

Check Each Corner

x	y	P
0	0	$.1 \cdot 0 + .12 \cdot 0 = 0$
18	0	$.1 \cdot 18 + .12 \cdot 0 = 1.8$
14.4	3.6	$.1 \cdot 14.4 + .12 \cdot 3.6 = 1.872$

1.872  
Max

Plain English recommendation:  
 Extend \$3.6 million in auto loans,  
 \$14.4 million in home loans,  
 & make profit of \$1.872.



12 (20 points) Read the whole problem before you start computing. Part of the problem has already been done for you!

A farmer has 150 acres of land suitable for growing corn and wheat. Cost to cultivate corn is \$40 per acre, cost to cultivate wheat is \$60 per acre. Farmer has \$7400 available for land cultivation. Each acre of corn requires 20 labor-hours and each acre of wheat requires 25 labor-hours. The farmer has a maximum of 3300 labor-hours. He expects to make \$150 per acre of corn and \$200 per acre of wheat.

The farmer wants to maximize his profit. He did some of the calculation already. He let  $x$  denote the number of acres of corn,  $y$  the number of acres of wheat. He let  $u$  denote the number of uncultivated acres,  $v$  the unused part of the budget,  $w$  the number of unused labor hours, and  $P$  denote the profit.

$x$	$y$	$u$	$v$	$w$	$P$	Constant
1	1	1	0	0	0	150
40	60	0	1	0	0	7400
20	25	0	0	1	0	3300
-40	-60	0	0	0	1	0

~~150~~  
~~123.33~~  
~~132~~

Help the farmer use the simplex algorithm to determine the maximum profit. To receive full credit, you must show all work.

(a) Show work for the simplex algorithm here. (use the back side of this page or the back side of the previous page if you need more space)

$R_2/60$

1	1	1	0	0	0	150
2/3	1	0	1/60	0	0	123.33
20	25	0	0	1	0	3300
-40	-60	0	0	0	1	0

$R_1 - R_2$

$\Rightarrow$

$R_3 - 25R_2$

$R_4 + 60R_2$

$x$	$y$	$u$	$v$	$w$	$P$	
1/3	0	1	-.0166	0	0	26.66
2/3	1	0	.0166	0	0	123.33
10/3	0	0	-.4166	1	0	216.67
0	0	0	1	0	1	7400

No more negatives DONE!

$x = 0$

$v = 0$

$u = 26.66$

$y = 123.33$

$w = 216.67$

$P = 7400$

(b) How many acres of corn and how many acres of wheat?

Farmer should grow 123.33 acres wheat & 0 acres corn.

(c) What is the maximum profit?

This generates \$7400 in profit.

(d) Are there any resources left over, and if so, how much of each?

26.66 unplanted acres, and 216.67 unused labor hours.

