Exam 3

MA 162: Finite Mathematics
University of Kentucky

November 25, 2013

Directions:

- Do not remove this page—you will turn in the entire exam.
- Complete this exam using only a pen or pencil and a simple calculator (not a cellphone).
- The point value for each question is shown in the exam booklet.
- On free response questions you must show all work in order to receive credit. Unjustified answers will receive no credit!
- If asked to explain, you must write clearly and in complete sentences.
- Use the back side of the exam for scrap work.
- It is not necessary to convert answers into decimal format. Your final answers may involve expressions like

$$\frac{271}{382}, \quad \binom{28}{5}, \quad 28!, \quad 5^{18},$$

- It is possible to earn more than 100 points on this exam!

Printed Name: Solutions **Section:** ________

Do Not Write Anything Here

Question	Points Possible	Score
Page 1	15 points	
Page 2	20 points	
Page 3	20 points	
Page 4	15 points	
Page 5	15 points	
Page 6	15 + 5 points	
Total	100 + 5 points	

Standard 52 card deck:

A♡	2♡	3♡	4♡	5♡	6♡	7♡	8♡	9♡	10♡	J♡	Q♡	K♡
A♢	2♢	3♢	4♢	5♢	6♢	7♢	8♢	9♢	10♢	J♢	Q♢	K♢
A♣	2♣	3♣	4♣	5♣	6♣	7♣	8♣	9♣	10♣	J♣	Q♣	K♣
A♠	2♠	3♠	4♠	5♠	6♠	7♠	8♠	9♠	10♠	J♠	Q♠	K♠

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1. (5 points) Your favorite fastfood restaurant offers super-value meals, which consist of a sandwich, a side, and a drink for $5. There are 4 kinds of sides. There are 5 kinds of drink. For the sandwich, you may choose from one of 4 kinds of hamburger, 3 kinds of chicken sandwich, or one kind of vegetarian sandwich. How many value meals are possible?

Sandwich choices = $4+3+1 = 8$
Drink Choices = 5
Side Choices = 4

Total $8 \cdot 5 \cdot 4$

2. (5 points) Bob has 1400 CDs in his CD collection. He is about to go on a very long drive, and wants to select 15 CDs to bring along for the trip. In how many ways can Bob select 15 CDs for his trip?

Choose 1400 from 15

$$\binom{1400}{15}$$

3. (5 points) LinkBlue asked Michael to change his password again (for the 100th time!). He can't remember his new password. He thought it was

$$ABY3B@333@Y$$

but that is not right. How many ways can he rearrange those characters to try another password?

A	1
B	2
Y	2
3	4
@	2
	11

$$\frac{11!}{1!\,2!\,2!\,4!\,2!}$$

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4. (15 points) A pair of fair six sided dice is rolled. The possible outcomes are listed below.

Sum ≤ 8

(1, 1)	(2, 1)	(3, 1)	(4, 1)	(5, 1)	(6, 1)
(1, 2)	(2, 2)	(3, 2)	(4, 2)	(5, 2)	(6, 2)
(1, 3)	(2, 3)	(3, 3)	(4, 3)	(5, 3)	(6, 3)
(1, 4)	(2, 4)	(3, 4)	(4, 4)	(5, 4)	(6, 4)
(1, 5)	(2, 5)	(3, 5)	(4, 5)	(5, 5)	(6, 5)
(1, 6)	(2, 6)	(3, 6)	(4, 6)	(5, 6)	(6, 6)

a.) Determine the event: One die has an even number and one die has the number 3. (By determine, I mean circle–or otherwise indicate–which of these simple events belong to the event.)

b.) A pair of fair six sided dice is rolled. Determine the probability that the values of the dice are different. (For example, a 2 and a 5, but not a 4 and a 4.)

6 of the 36 are "same", so

$$\frac{30}{36}$$

c.) A pair of fair six sided dice is rolled. Determine the probability that the sum of the value of the dice is at most 8.

$$P(\leq 8) = P(8) + P(7) + P(6) + \cdots + P(2) = \frac{26}{36}$$

5. (5 points) 200 students were surveyed regarding the number of credit hours they are taking in the current semester. Here are their responses:

Credit hours	12 or fewer	13 hours	14 hours	15 hours	16 hours	17 hours	18 or more
Number students	46	51	31	55	10	6	1

A student is selected at random from this group. What is the probability that they are taking at least 15 credit hours?

$$P(\geq 15) = P(15) + P(16) + P(17) + P(\geq 18)$$

$$= \frac{55 + 10 + 6 + 1}{200} = \frac{72}{200}$$

6. (10 points) A fair coin is flipped 8 times.

a.) What is the probability of flipping heads on the third flip, given that the first two flips were tails?

$\frac{1}{2}$; Coin flips are independent

b.) What is the probability of flipping at least 2 tails? $= 1 - P(\text{No more than } 1)$

$= 1 - P(0\,T) - P(1\,T)$

$= 1 - \frac{\binom{8}{0}}{2^8} - \frac{\binom{8}{1}}{2^8}$

7. (10 points) Kyle wants to arrange his DVD collection on his bookshelf. He has the following movies: 6 science-fiction movies, 2 ganster movies, 3 superhero movies, and 3 fantasy movies.

a.) How many ways can the DVDs be ordered on his shelf, if there is no restriction on how the movies can be placed?

14 movies

$14!$

b.) Kyle wants to place all of the science fiction movies together, all of the superhero movies together, all of the fantasy movies together, and the two ganster movies together. How many ways can he do this?

Sci-Fi	Fant	Gangster	Super Hero
$6!$	$3!$	$2!$	$3!$

$4!$

$4! \cdot 6!\, 3!\, 2!\, 3!$

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8. (15 points) The NFL has received a lot of bad press in the last few years, as more and more retired players are reporting serious health problems due to concussions. A group of 100 men in a neigborhood were surveyed regarding their opinion on increasing safety standards for high school football players. The columns in the table are as follows:

- Column 1: The man had played football and he has a son that plays football.
- Column 2: The man had played football but none of his sons play football.
- Column 3: The man did not play football, but he has a son that plays football.
- Column 4: The man did not play football and none of his sons play football.

	Column 1	Column 2	Column 3	Column 4	Total
Supports increased safety standards	12	6	15	8	41
Opposed increased safety standards	8	14	8	5	35
No opinion	2	2	2	18	24
Total	22	22	25	31	100

a.) What is the probability that a randomly selected man supports increased safety standards?

$$\frac{41}{100} = P(\text{Approve Safety})$$

b.) What is the probability that a randomly selected man supports increased safety standards, given that he has a son that plays football?

Look at Columns 1 + 3

$$\frac{12+15}{22+25} = \frac{27}{47} = P(\text{Approve Safety} \mid \text{Son Plays})$$

c.) Based on your answers to the above parts, are the events "supports increased safety standards" and "has son that plays football" independent? Explain. (For the sake of this problem, assume that two probabilities that agree up to ± 0.01 are "the same")

If independent, the

$$P(\text{Approve Safety} \mid \text{Son Plays}) = P(\text{Approve Safety})$$

But $\frac{27}{47} \neq \frac{41}{100}$

So not independent

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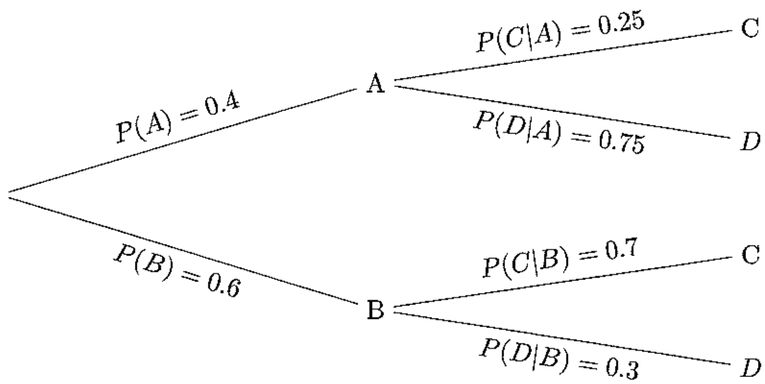
9. (15 points) The accompanying tree represents a two stage experiment.



$= .4 \cdot .25 = .1$

$= .42$

a.) Determine $P(C|B)P(B)$

$= 0.6 \cdot .7 = 0.42$

b.) Determine $P(C)$

$P(C|B)P(B) + P(C|A)P(A)$

$= 0.42 + 0.1 = 0.52$

Baye's Theorem

c.) Determine $P(A|C)$

$$= \frac{P(C|A)P(A)}{P(C)} = \frac{0.1}{0.52}$$

$= .192307$

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10. (10 points) Two cards are drawn from a well shuffled deck of 52 cards.
   a.) Determine the probability that both cards are Kings.

See next page

   b.) Determine the probability that the cards are of two different suits.

11. (10 points) Your dresser drawer has eight white socks, six black socks, and five plaid socks.
   a.) You reach in and pull out two socks. What is the probability that you have a matching pair?

   b.) You reach in and pull out three socks. What is the probability that you can form a matching pair from these three socks?

Alternate approaches

#10 & #11 can be approached from many different points of view. These approaches are all correct.

10 (a) (i) $\dfrac{\binom{4}{2}}{\binom{52}{2}} = 0.0045\ldots$

choose 2 of the 4 Kings

choose 2 of 52 cards

(Note: Should not include $\binom{13}{1}$, as the Kind (Rank) has already been chosen)

(ii) $\underbrace{\dfrac{4}{52}}_{} \cdot \underbrace{\dfrac{3}{51}}_{} = 0.0045$

On first draw, have 52 cards, 4 are Kings | On second draw, have 51 cards & 3 are Kings

(iii) $1 - P\left((\text{Both cards Kings})^c\right)$

Most people that attempted the problem this way failed to interpret $(\text{Both cards Kings})^c$ correctly.

The opposite of (Both are Kings) is (No more than one is a King)

$$1 - \left(\frac{\binom{4}{1}\binom{48}{1}}{\binom{52}{2}} + \frac{\binom{48}{2}}{\binom{52}{2}} \right) = 0.0045$$

choose one King & one non-King | choose 2 non-Kings

10 (b)

(i) Fastest method: Choose anything for the first card.
For the second card, there are 51 left,
39 are of different suits than the first card.

So $$\frac{39}{51} = 0.7647$$

(ii) $$\frac{\binom{4}{2}\binom{13}{1}\binom{13}{1}}{\binom{52}{2}} = 0.7647$$

For each of the 2 suits, choose rank.

Choose 2 of the four suits.

(iii) $1 - P(\text{same suit})$

$$= 1 - \frac{\binom{4}{1}\binom{13}{2}}{\binom{52}{2}} = 0.7647$$

Choose the suit

Choose 2 of the 13 cards of that suit

11 (a) Pairs: WW, BB, or PP

WW: $\dfrac{\binom{8}{2}}{\binom{19}{2}}$ BB $\dfrac{\binom{6}{2}}{\binom{19}{2}}$ PP $\dfrac{\binom{5}{2}}{\binom{19}{2}}$

So $$\frac{\binom{8}{2}+\binom{6}{2}+\binom{5}{2}}{\binom{19}{2}} = 0.3099$$

(b)

(i) Fastest way

$1 - P(\text{No match})$

Only way to have no match is if we draw one of each type of sock.

$$1 - \frac{\binom{8}{1}\cdot\binom{6}{1}\cdot\binom{5}{1}}{\binom{19}{3}} = 0.7523$$

(ii) Long Way:

$$\frac{\overset{WWW}{\binom{8}{3}} + \overset{BBB}{\binom{6}{3}} + \overset{PPP}{\binom{5}{3}} + \overset{WW\ W^c}{\binom{8}{2}\binom{11}{1}} + \overset{BB\ B^c}{\binom{6}{2}\binom{13}{1}} + \overset{PP\ P^c}{\binom{5}{2}\binom{14}{1}}}{\binom{19}{3}}$$

$= 0.7523$

A lot of people either only included the first three terms, or they only included the last three terms.