

# MA162: Finite mathematics

Paul Koester

University of Kentucky

December 9, 2013

*Solutions*

## SCHEDULE:

- Web Assign assignment (Chapter 5.2) due on Tuesday, December 10 by 6:00 pm.
- Web Assign assignment (Chapter 5.3) due on Friday, December 13 by 6:00 pm.
- Exam 4 on Monday, December 16, 8:30 pm to 10:30 pm.

Today is Chapter 5.3: Amortized Loans

# Annuity Formulas

The future value of a simple ordinary annuity with  $n$  level payments of  $R$  dollars each period, paid at the end of each period into an account that earns interest at the rate of  $i$  per period is

$$S = R \frac{(1+i)^n - 1}{i} = Rs_{\overline{n}|i}$$

The present value of this annuity is

$$P = R \frac{1 - (1+i)^{-n}}{i} = Ra_{\overline{n}|i}$$

# Amortized Loans

This final section is concerned with loans. There are two main differences between the section and the previous.

- For a loan, we take the point of view of the borrower. Thus, for a loan, we assume the person pays regular cash flows. In an annuity, we assume the person receives regular cash flows.
- For a loan, we usually assume that the total amount (either in present value or future value) is known, and we usually wish to determine the size of the regular payments.

# Loan Formulas

Consider a loan with  $n$  level payments of  $R$  dollars each period, paid at the end of each period, and suppose the loan accumulates interest at the rate of  $i$  per period. Let  $P$  denote the principal (present value) of the loan and let  $S$  denote the future value of the loan. To determine the payment size,  $R$ , we can use

$$R = \frac{Si}{(1+i)^n - 1} = \frac{S}{s_{\overline{n}|i}}$$

OR

$$R = \frac{Pi}{1 - (1+i)^{-n}} = \frac{P}{a_{\overline{n}|i}}$$

# Loan Example 1

You wish to buy a new car for  $\overbrace{\$13,000}^{\text{Present Value}}$ . The car dealer offers you a 36 month loan where the interest is 2.4% compounded monthly. Assume the loan payments are to be made at the end of each month.

- Determine the size of your monthly payment.
- Determine the interest charges (look at the difference between what you borrowed and what you paid back)

$$\begin{aligned} m &= 12 \\ n &= 36 \\ r &= 0.024 \\ i &= 0.024/12 \\ &= 0.002 \end{aligned}$$

$$13,000 = R a_{\overline{36}|0.002} \implies R = \frac{13000}{a_{\overline{36}|0.002}} = \boxed{\$374.63}$$

Monthly Payment

$$36 \text{ payments @ } 374.63 \implies \text{Pay back } \$13,486.61$$

$$\text{Interest charge was } 13486.61 - 13000.00 = \boxed{486.61}$$

## Loan Example 2

You wish to buy a new home. The home costs \$250,000. Your bank requires you to make a down payment of 20% of the value of the home. You will take out a loan for the remaining value of the home. The bank offers you 30 year loan where the interest rate is 7.0% per year, compounded monthly.

- Determine the size of your house payments.
- Determine the interest charges (look at the difference between what you borrowed and what you paid back)

Down Payment  $0.2 \times 250,000 = 50,000$ , you will then borrow \$200,000

$$200,000 = R a_{\overline{360}|0.07/12} = R a_{\overline{360}|0.005833}$$

$$\text{So } R = \frac{200,000}{a_{\overline{360}|0.005833}} = \$1,330.60 \leftarrow \text{Monthly Payment.}$$

Paid back  $1,330.60 \times 360 = 479,017.80$

So  $279,017.80$  paid back in interest

$$\begin{aligned} m &= 12 \\ n &= 12 \times 30 \\ &= 360 \\ r &= 0.07 \\ i &= \frac{0.07}{12} \end{aligned}$$

# A Savings Example

Donna wishes to save a total of  $\$20,000$  <sup>FV</sup> over the next 2 years. She plans on making deposits into a savings account at the end of each quarter for the next 8 quarters. The savings account pays 1.6% nominal interest per year, compounded quarterly. Determine the required size of her quarterly payments.

$$t = 2$$
$$m = 4$$
$$n = 4 \cdot 2 = 8$$
$$r = .016$$
$$i = \frac{.016}{4} = 0.004$$

$$20000 = R \cdot s_{\overline{8}|0.004} \quad \text{so} \quad R = \frac{20000}{s_{\overline{8}|0.004}}$$

$$s_{\overline{8}|0.004} = \frac{(1.004)^8 - 1}{.004} = 8.1129$$

$$\text{so} \quad R = \frac{20000}{8.1129} = \boxed{2465.21}$$

Quarterly Payment.