MA162: Finite mathematics

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November 4, 2013

Schedule:

- Web Assign assignment (Chapter 6.3) due on Tuesday, November 5 by 6:00 pm.
- Web Assign assignment (Chapter 6.4) due on Friday, November 8 by 6:00 pm.
- Exam 3 on Monday, November 25, 5:00 pm to 7:00 pm.

Today we look at Chapter 6.4

6.4: Permutations

- How many different ways can the letters O, P, and E be re-arranged?

6.4: Another Permutation

- Bob, Jane, Mike, and Sally are elected to serve on a committee.

- One will have to serve as president, one as vice-president, one as treasurer, and one as secretary.

- In how many ways can the positions be assigned to Bob, Jane, Mike, and Sally?

6.4: Permutations: More formally

- Given a set of distinct objects, a permutation of this set is an arrangement which specifies an ordering on the elements of the set.

- The number of permutations of a set of n distinct items is

$$n \cdot (n-1) \cdot (n-2) \cdots 3 \cdot 2 \cdot 1$$

- The above expression is called n factorial and is denoted $n!$.

6.4: Permutations: More Generally

- Bob, Jane, Mike, Sally, Ronny, and Kenny are elected to serve on a committee.

- One will have to serve as president, one as vice-president, one as treasurer, and one as secretary. Two don't have to do anything extra.

- In how many ways can the 4 positions be assigned to Bob, Jane, Mike, Sally, Ronny, and Kenny?

6.4: Permutations: More formally

- Given a set of distinct objects, a permutation of this set is taken r at a time is an arrangement which chooses r of the elements of the set and specifies an ordering of these r items..

- The number of permutations of a set of n distinct items taken r at a time is
$$P(n, r) = \frac{n!}{(n-r)!}$$

6.4: Permutations: Not Necessarily Distinct Items

- You have a group of 3 boys and 4 girls.
- The are to stand in line.
- How many arrangements are there?

6.4: Permutations: Not Necessarily Distinct Items

- You have a set with n items.

- n_1 of the items are of alike of one kind. n_2 items are alike and of another kind. ... n_r items are alike of yet another kind.

- All of the items are one of the r types, so that

$$n_1 + n_2 + \ldots + n_r = n$$

- The number of these n objects taken n at a time is then

$$\frac{n!}{n_1! n_2! \ldots n_r!}$$

6.4: Permutations: Rearranging Letters

- In how many ways can the letters in KENTUCKY be rearranged?

- A poor musician has forgetten how to play her favorite melody. She thought it was AACEGEA, but that doesn't sound right. She knows that she has the right notes, and the right number of repeats of each note, She's just putting them in the wrong order. In how many ways can she rearrange those notes?

6.4: Combinations - when order doesn't matter

- There are 10 new songs you'd like to download.
- You just received a gift card which will allow you to download 7 songs.
- In how many ways can you choose 7 out of these 10 songs?
- NOTE: The order in which you download the songs does not matter!

6.4: Combinations - in general

- A combination of n distinct items taken r at a time is an un-ordered selection of r of these n items.

- The number of such combinations is

$$\binom{n}{r} = C(n, r) = \frac{n!}{r!(n-r)!}$$

- The expression $\binom{n}{r}$ is pronounced "n choose r." This is also called a "binomial coefficient."

6.4: Combinations

- You walk home from school everyday.
- Home is 3 blocks south and 2 blocks east from school.
- To keep your walk interesting, you try to take a different path every day.
- You also want to get home quickly, so you will never turn north, and you will never turn west.
- How many different paths can you take?

6.4: Counting Cards

- A standard deck of cards has 52 cards.

- A standard Poker hand has 5 of these cards.

- How many 5 card hands are possible?

6.4: Counting Cards

- A standard deck of cards has the following cards:
  A♡ 2♡ 3♡ 4♡ 5♡ 6♡ 7♡ 8♡ 9♡ 10♡ J♡ Q♡ K♡
  A♢ 2♢ 3♢ 4♢ 5♢ 6♢ 7♢ 8♢ 9♢ 10♢ J♢ Q♢ K♢
  A♣ 2♣ 3♣ 4♣ 5♣ 6♣ 7♣ 8♣ 9♣ 10♣ J♣ Q♣ K♣
  A♠ 2♠ 3♠ 4♠ 5♠ 6♠ 7♠ 8♠ 9♠ 10♠ J♠ Q♠ K♠
- See
  http://en.wikipedia.org/wiki/List_of_poker_hands
  for definitions of poker hands.
- Try counting the number of each type of poker hand yourself. This is a great way to practice the ideas from this chapter.
- Check your work against wikipedia's, but be warned that not everything you read on *wikipedia* is reliable!

6.4: Full house or 4 of a kind?

- A full house is a hand with 3 of one kind and 2 of another kind.

- A four of a kind is a hand with 4 of one kind, and some other card.

- Which is more common? Full house or 4 of a kind?

6.4: Two pair or 3 of a kind?

- A Two pair consists of two cards of one kind, two cards of different kind, and a fifth card, whose kind does not match either of the pairs.

- For example, A♡ A♠ 5♣ 5♡ 7♡

- A Three of a kind consists of three cards of one kind, and two other cards, in which the other two cards do not form a pair, nor do either of kinds match the three of a kind.

- For example, A♡ A♠ A♢ 5♣ 7♠

- Which is more common? Two pair or 3 of a kind?

6.4: Flush or Straight?

- A full house is a hand with 3 of one kind and 2 of another kind.

- A straight is a hand with 5 cards in sequence, but not all of the same suit.

- For example, 8♡ 9♠ 10♣ J♡ 7Q♡ Note: an ACE can go before 1 or after King.

- Which is more common? Full house or a straight (but not straight flush)?

6.4: Counting Cards

- A hand in 7 card stud consists of 7 cards, in which 3 cards are face-up (visible to all players) and 4 cards are face-down.

- How manay distinct hands can be dealt to a player in 7 card stud?

- NOTE: It is NOT $\binom{52}{7}$.