MA162: Finite mathematics

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SCHEDULE:

- Web Assign assignment (Chapter 6.4) due on Friday, November 8 by 6:00 pm.
- Web Assign assignment (Chapter 7.1, 7.2) due on Tuesday, November 12 by 6:00 pm.
- Exam 3 on Monday, November 25, 5:00 pm to 7:00 pm.

Today we introduce Probability Theory

7.1: Probability Theory Basics

- There is much overlap between Counting (Chapter 6) and Probability Theory (Chapter 7), but some terminology is different
- An experiment is an activity with observable results
- A sample point is an outcome of an experiment
- A sample space is the collection of all sample points of an experiment
- An event is a collection of some (possibly all, possibly none) sample points of an experiment
- A simple event is an event containing exactly one sample point

7.1: Counting to Probability Dictionary

Counting		**Probability**
element of a set	$\leftrightarrow$	sample point
universal set	$\leftrightarrow$	sample space
subset of a set	$\leftrightarrow$	event

7.1: Example of a sample space

- The experiment: You flip a quarter and a dime, and observe whether heads or tails on each coin.
- What is the sample space?

set of ordered pairs of H & T

$\{(H,H), (H,T), (T,H), (T,T)\}$

- What are the simple events?

(H,H)

(H,T)

(T,H)

(T,T)

7.1: More than one space?

- The experiment: You flip two quarters, and observe whether heads or tails is on each coin.
- Describe one sample space:

{ (H, H), (H, T), (T, H), (T, T) }

- Describe another sample space:

{ two heads, one heads & one tails, two tails }

NOTE: The first sample space has 4 simple events, the second has 3. But they both describe same experiment!

7.1: Probability Spaces

Students in MA 162 are asked the following questions:

F, S
Jr, Sr

A, C,
W, N

- What year of college? Freshman, Sophomore, Junior, Senior
- Have you completed FIN 300? Already completed, Currently taking, Will take in future, No plans to take
- Describe a sample space for this experiment:

{(F, A), (F, C), (F, W), (F, N), (S, A), (S, C), (S, W), (S, N),
(Jr, A), (Jr, C), (Jr, W), (Jr, N), (Sr, A), (Sr, C), (Sr, W), (Sr, N)}

- Determine the event: Student is a Junior

{(Jr, A), (Jr, C), (Jr, W), (Jr, N)}

- Determine the event: Student has not started FIN 300

{(F, W), (F, N), (S, W), (S, N), (Jr, W), (Jr, N), (Sr, W), (Sr, N)}

- Determine the event: Student is Sophomore or Junior and has already completed FIN 300

{(S, A), (Jr, A)}

7.1: Mutually Exclusive Events

- Two events are Mutually Exclusive if they cannot occur simultaneously
- In other words, $E_1 \cap E_2 = \emptyset$
- Are the events: "Student is Junior" and "Student is Senior" mutually exclusive?

  Yes Student can't be Junior & Senior at same time.

- Are the events: "Student is Junior" and "Student has already completed FIN 300" mutually exclusive?

  No (Jr, A) is in both events

7.2: Definition of Probability

- The Probability of an event is a number between 0 and 1
- The larger the probability, the more likely that outcome
- Probability of 0 means outcome will NEVER happen
- Probability of 1 means outcome will ALWAYS happen

7.2: Probability Measure

- Consider an experiment with sample space $S = \{s_1, s_2, \ldots, s_n\}$.

- A probability function, P, assigns a to each event a number between 0 and 1 according to the rules:

  - $0 \leq P(E) \leq 1$ for each event E.

  - $P(s_1) + P(s_2) + \ldots + P(s_n) = 1$

  - If events E_1 and E_2 are mutually exclusive then

$$P(E_1 \cup E_2) = P(E_1) + P(E_2)$$

7.2: Emperical Distibution: Example

- 200 MA 162 students are polled and asked: "How many core business courses have you completed?"
- Sample space: {None, One, Two, ...}

Here are the results:

	None	One	Two	Three	At least Four
Number	5	23	42	63	67

- What is probability student selected at random has completed exactly one business course?
- What is probability student has completed no more than two business courses?

7.2: Another Emperical Distribution

University administrators polled a group of 500 students who had withdrawn from at least one course in the previous semester. Students were asked for the primary reason for withrdrawing: "fell behind due to illness", "fell behind due to being enrolled in too many course", "fell behind due to beign too busy at work", "felt unprepared and withdrew to take a prereq", "other reasons"

7.2: Another Emperical Distribution

Reason	Frequency	Probability
Illness	153	153/500
Too many courses	268	268/500
Too busy at work	42	42/500
Unprepared	17	17/500
Other	20	20/500
Total	500	

- Fill in the rest of the table
- Given a random student who withdrew from a course last semester, what is the probability this student withdrew for a reason other than illness? $\frac{500-153}{500} = 0.694$
- Given a random student who withdrew from a course last semester, what is the probability this student withdrew due to being too busy, either with other courses or with work?

$$\frac{268+42}{500} = \frac{310}{500} = 0.62$$