MA162: Finite mathematics

Paul Koester

University of Kentucky

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SCHEDULE:

- Web Assign assignment (Chapter 7.1, 7.2) due on Tuesday, November 12 by 6:00 pm.
- Web Assign assignment (Chapter 7.3) due on Friday, November 15 by 6:00 pm.
- Exam 3 on Monday, November 25, 5:00 pm to 7:00 pm.

Today is Chapter 7.3. We learn some general techniques for computing probabilities

7.3: Some Rules for Computing Probabilities

- $0 \leq P(E) \leq 1$ for any event E
- $P(S) = 1$ where S is the entire sample space
- $P(E \cup F) = P(E) + P(F) - P(E \cap F)$
- Notice that if E and F are mutually exclusive, then

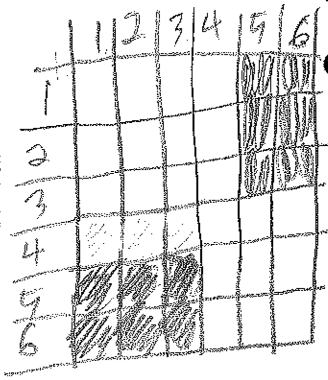
$$P(E \cup F) = P(E) + P(F)$$

- $P(E^c) = 1 - P(E)$
- For any two events E and F,

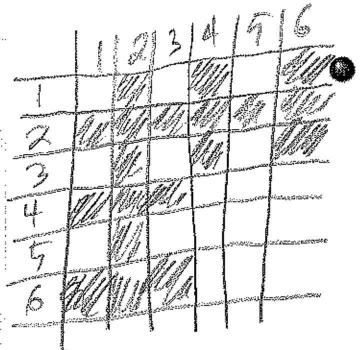
$$P(E) = P(E \cap F) + P(E - F)$$

7.3: Dice

A pair of fair 6 sided dice is rolled.



- Determine the probability that one die turns up a number not greater than 3 and the other die turns up a number greater than 4. $12/36 = \frac{1}{3}$



- Determine the probability that one die turns up a number not greater than 3 and one die turns up an even number.

$19/36$

- Determine the probability that the sum of the values of the dice is at least 8.

	1	2	3	4	5	6
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10
5	6	7	8	9	10	11
6	7	8	9	10	11	12

$\frac{15}{36}$

7.3: Flipping coins

A fair coin is flipped 5 times.

- What is the probability of turning up exactly two heads?

  _ _ _ _ _ $\binom{5}{2}\left(\frac{1}{2}\right)^5 = \frac{5 \cdot 4}{2} \cdot \left(\frac{1}{2}\right)^5 = 0.3125$

- What is the probability of turning up at least two heads?

  Hard way: At least 2
  $\Longrightarrow$ Exactly 2 $\cup$ Exactly 3 $\cup$ Exactly 4 $\cup$ Exactly 5

  Easier: At least 2
  $\Longrightarrow$ Opposite of $< 2 \Rightarrow$ Opposite of (None $\cup$ Exactly 1).

  Now, None: $\left(\frac{1}{2}\right)^5$, Exactly 1: $\binom{5}{1} \cdot \left(\frac{1}{2}\right)^5$

  So, want $1 - \left(\left(\frac{1}{2}\right)^5 + \binom{5}{1}\left(\frac{1}{2}\right)^5\right) = 0.8125$

7.3: Drawing Cards

Doesn't matter what first card is. Second card must be one of the 12 matching suit cards

Two cards are drawn from a standard deck of 52 cards.

- What is the probability the two cards have the same suit?

$$\frac{\binom{12}{1}}{\binom{52}{2}} = \frac{12}{\frac{52\cdot 51}{2}}$$

Total # 2 card hands

- What is the probability that one card has a number between 3 and 7?

32 cards are not between 3 & 7.

$$1 - P\left(\begin{smallmatrix}\text{Neither between}\\ 3 \,\&\, 7\end{smallmatrix}\right) = 1 - \frac{\binom{32}{2}}{\binom{52}{2}} = 1 - \frac{32\cdot 31}{52\cdot 51}$$

- What is the probability that the two card hand contains a King and a Spade?

Be careful with double counting the King of Spades! → Next Page

- What is the probability that the two card hand contains a King or a spade?

$$1 - P\left(\begin{smallmatrix}\text{No (King}\\ \text{or Spade)}\end{smallmatrix}\right) = 1 - \frac{\binom{36}{2}}{\binom{52}{2}} = 1 - \frac{36\cdot 35}{52\cdot 51}$$

36 cards neither K nor Spade

$$P(\text{King} \,\&\, \text{Spade}) = P(\text{Hand has King}) + P(\text{Hand Has Spade}) - P(\text{Hand has King OR Spade})$$

48 non-King

$$\text{But } \quad P(\text{King}) = 1 - P(\text{No King}) = 1 - \frac{\binom{48}{2}}{\binom{52}{2}}$$

39 non-Spade.

$$P(\text{Spade}) = 1 - P(\text{No Spade}) = 1 - \frac{\binom{39}{2}}{\binom{52}{2}}$$

$$P(\text{King OR Spade}) = 1 - P(\text{No King And No Spade})$$

$$= 1 - \frac{\binom{36}{2}}{\binom{52}{2}}$$

$$\text{So } \quad P(\text{King} + \text{Spade}) = \left(1 - \frac{\binom{48}{2}}{\binom{52}{2}}\right) + \left(1 - \frac{\binom{39}{2}}{\binom{52}{2}}\right) - \left(1 - \frac{\binom{36}{2}}{\binom{52}{2}}\right)$$

$$= 1 + \frac{\binom{36}{2}}{\binom{52}{2}} - \frac{\binom{48}{2}}{\binom{52}{2}} - \frac{\binom{39}{2}}{\binom{52}{2}} = 1 + \frac{\frac{36\cdot 35}{2} - \frac{48\cdot 47}{2} - \frac{39\cdot 38}{2}}{\frac{52\cdot 51}{2}}$$

$$= 1 + \frac{36\cdot 35 - 48\cdot 47 - 39\cdot 38}{52\cdot 51} = 1 - \frac{2478}{2652} = 0.065611$$

7.3: Another Probability Rule

- E is an event

- $F_1, F_2, \ldots, F_r$ are events satisfying

  - $S = F_1 \cup F_2 \cup \cdots \cup F_r$
  - Each pair F_i and F_j are mutually exclusive, for $i \neq j$

- Then $P(E) = P(E \cap F_1) + P(E \cap F_2) + \cdots + P(E \cap F_r)$

7.3: More dice

A six sided die is rolled 4 times. What is the probability of rolling at least one six?

$$1 - P(\text{No Six}) = 1 - \frac{5 \cdot 5 \cdot 5 \cdot 5}{6 \cdot 6 \cdot 6 \cdot 6}$$

$$= 0.517746\ldots$$

7.3: Gaming

- A group of 243 video gamers were asked about video game habits.
- Question 1: Which game consoles do you own? “Own PS 3”, “Own X-Box 360”, “Own neither”, “Own both”
- Question 2: How many hours per week do you play? “no more than 2 hours”, “between 2 to 6 hours”, “more than 6 hours”
- Results are recorded on next page

7.3: Gaming

	PS 3 only	X-Box only	Both	Neither	Total
$<$ 2 hours	47	23	7	17	94
2 to 6 hours	34	41	11	3	89
$>$ 6 hours	15	18	25	2	60
total	96	82	43	22	243

- Probability random gamer owns an X-box but not a PS-3?

  $82/243$

- Probability random gamer owns a PS-3?

  $(96+43)/243 = 139/243$

- Probability random gamer plays at least 2 hours per week?

  $(89+60)/243 = 149/243$

- Probability random gamer owns an X-box and plays two hours or less per week? $23/243$

Conditional Gaming

	PS 3 only	X-Box only	Both	Neither	Total
$<$ 2 hours	47	23	7	17	94
2 to 6 hours	34	41	11	3	89
$>$ 6 hours	15	18	25	2	60
total	96	82	43	22	243

- Restrict attention only to gamers who own both consoles. What is probability they play at least two hours per week?

$$\frac{11+25}{43} = \frac{36}{43}$$

- Restrict attention to gamers who play less than two hours per week. What is probability they own a PS 3?

$$\frac{47+7}{94} = \frac{54}{94}$$