MA162: Finite mathematics

Paul Koester

University of Kentucky

With Correction!

November 13, 2013

SCHEDULE:

- Web Assign assignment (Chapter 7.3) due on Friday, November 15 by 6:00 pm.
- Web Assign assignment (Chapter 7.4) due on Tuesday, November 19 by 6:00 pm.
- Exam 3 on Monday, November 25, 5:00 pm to 7:00 pm.

Today is Chapter 7.4. We will see how some of the techniques from Chapter 6 can be applied to probability problems.

7.4: Counting Techniques in Probability Theory

- To find the probability of an event E, we can count the number of simple events in E, $n(E)$
- then count the number of simple events in the sample space, $n(S)$
- then divide:

$$P(E) = \frac{n(E)}{n(S)}$$

- Computing $n(E)$ and $n(S)$ may require techniques from Chapter 6.

7.4: Counting Cards

- A 5 card hand is drawn from a standard deck of 52 cards.
- Determine the probability of drawing a "4 of a kind"

- Begin by counting the total number of possible hands

$$\binom{52}{5}$$

- Then count the number of ways to get "4 of a kind."

$$\binom{13}{1}\cdot\binom{4}{4}\cdot\binom{12}{1}\cdot\binom{4}{1} = 13\cdot 1\cdot 12\cdot 4 = 624$$

So,

$$\frac{624}{\binom{52}{5}}$$

7.4: Multiple Guess

$\frac{\binom{20}{5} 4^{15}}{5^{20}} \approx 0.1745$

- An MA 123 exam consists of 20 questions, multiple choice
- There are 5 choices for each question
- A student randomly guesses on all of the questions.
- What is probability student gets exactly 5 of the questions correct?

#Ways to answer all 20 questions $= 5^{20}$ (5 choices for each of 20 questions, answers are independent)

#Ways to answer 5 correct + 15 incorrect: $\binom{20}{5} \cdot 1^5 \cdot 4^{15}$

($\binom{20}{5}$: which 5 are correct; 1^5: 1 choice for each of 5 correct; 4^{15}: 4 choices for each of the 15 incorrect)

- What is the probability this student gets at least an 80% on the exam? $\Rightarrow$ 16 or 17, or 18 or 19 or 20 correct

$$\frac{\binom{20}{16}4^4 + \binom{20}{17}4^3 + \binom{20}{18}4^2 + \binom{20}{19}4 + \binom{20}{20}}{5^4} \approx 1.38 \times 10^{-8}$$

7.4: Urns

- A bag contains 5 white balls, 12 red balls, and 9 green balls.
- You reach in and draw out 5 balls.
- What is the probability that 3 of the balls are red and 2 are green?

$$\frac{\binom{5}{0}\binom{12}{3}\binom{9}{2}}{\binom{5+12+9}{5}} = \frac{\frac{12\cdot 11\cdot 10}{3\cdot 2\cdot 1}\cdot\frac{9\cdot 8}{2}}{\frac{26\cdot 25\cdot 24\cdot 23\cdot 22}{5\cdot 4\cdot 3\cdot 2}}$$

$$= \frac{7920}{240230} = 0.0344$$

7.4: Maybe Tonight?

- A dish contains candy hearts, one out of eight of the hearts has the logo "Maybe Tonight?"
- George Michael keeps picking pieces of candy at random from a bowl of 200 until he picks a "Maybe Tonight?" piece. He stops once he picks such a piece.
- What is the probability that he picks his first "Maybe Tonight?" piece on his 4th pick?

without replacement ⇒ He does not put pieces back in

$$\frac{175}{200} \cdot \frac{174}{199} \cdot \frac{173}{198} \cdot \frac{25}{197}$$

First pick there are 200 pieces, 175 non "Maybe tonight"s

Second pick, one of the non-"Maybe tonights" is gone.

Now 2 of the non-"maybe tonights" are gone

Now 3 pieces are gone, 25 "Maybe tonights" remain.

7.4: What's Broke?

- A math professor using a computer system to print barcodes on exam papers, and, after students take the exam, the exam papers are scanned and graded from a computer screen.
- 840 pages are printed in "version A" (120 exams, 7 pages each)
- 840 pages are printed in "version B" (120 exams, 7 pages each)
- The duplication company printed the different versions on different printers.
- The exams are shuffled before they are given to students.
- By the time the exams are scanned, the "version A"s and "version B"s are well mixed.

7.4: What's Broke?

- The computer was unable to read the barcodes on 100 pages, these had to be corrected manually.
- It just so happened that every misread barcode was from the "version A."
- The math professor thinks there is a problem with one of the duplication company's printers.
- The duplication company claims that their printers are fine, the problem is with the math department's scanner.
- Who is right?

Duplication Company's claim is that by chance, 100 version A & no version B's were misread.

Probability of this, given that 100 of the 1680 were misread is

$$\frac{\binom{840}{100}\binom{840}{0}}{\binom{1680}{100}} \approx 0.000000000000000000000000000343499 = 3.44 \times 10^{-32}.$$

It seems pretty far fetched to claim that the scanner just happened to misread 100 version A's and no version B's.

Just to get an idea of how crazily small this probability is:

Is it trustworthy??

Some guy on the internet estimated that there are 10^{26} grains of sand in the Sahara desert. Suppose one were to add a single grain of sugar to the Sahara desert, then mix the sand & the sugar. Now you select one grain at random.

$P(\text{sugar}) = \frac{1}{10^{26}}$, but this is still ≈ 3000 times more likely than the scanner randomly misreading 100 version A and no version B papers