# MA162: Finite mathematics

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SCHEDULE:

- Web Assign assignment (Chapter 7.5 and 7.6) due on Friday, November 22 by 6:00 pm.
- Exam 3 on Monday, November 25, 5:00 pm to 7:00 pm.

Today is Chapter 7.6, Baye's Theorem

# 7.6: Lung cancer and smoking

Facts taken from `http://lungcancer.about.com/od/Lung-Cancer-And-Smoking/f/Smokers-Lung-Cancer.htm`

- 0.2% probability of lung cancer for men who never smoked
- 5.5% probability of lung cancer for male former smokers
- 15.9% probability of lung cancer for current male smokers
- 24.4% probability of lung cancer for male "heavy smokers" defined as smoking more than 5 cigarettes per day

A sample of 10,000 men.

- 5680 of these men never smoked
- 2132 are former smokers
- 1737 are current "light" smokers
- The remaining men are "heavy" smokers.

What is the probability a randomly chosen male from this group will develop lung cancer?

L = Develop Lung Cancer

N = Never Smoked

F = Former Smokers

C = Current Smokers

H = Heavy Smokers

$P(N) = \frac{5680}{10000}$

$P(F) = \frac{2132}{10000}$

$P(C) = \frac{1737}{10000}$

$P(H) = \frac{451}{10000}$

$$P(L) = P(L|N)\cdot P(N) + P(L|F)P(F) + P(L|C)P(C) + P(L|H)P(H)$$

$$= 0.002\cdot\frac{5680}{10000} + 0.055\,\frac{2132}{10000} + 0.159\cdot\frac{1737}{10000} + 0.244\cdot\frac{451}{10000}$$

$$= 0.0514847 = 5.15\%$$

# 7.6: Lung cancer and smoking

A randomly chosen male from this group is chosen. He has lung cancer.

- What is the probability that he is heavy smoker?

$$P(H|L) = \frac{P(H\cap L)}{P(L)} = \frac{P(L|H)\,P(H)}{P(L)} = \frac{0.244 \cdot .0451}{0.05148} = 0.21376$$

21.4%

Compare if male is chosen @ random, there is 4.5% they are heavy smoker.

- What is the probability that he is either a light smoker or a former smoker?

$$P(C\cup F|L) = \underbrace{P(C|L) + P(F|L)}_{\text{Mut. Exc.}} = \frac{P(L|C)\,P(C)}{P(L)} + \frac{P(L|F)\cdot P(F)}{P(L)}$$

$$= \frac{.159 \cdot .1737 + .055 \cdot .2132}{.05148}$$

$$= 0.7642 = 76\%$$

## 7.6: Battery Ram

You have two boxes of batteries. One box is supposed to contain new batteries. You put dead batteries in the other box.
You just realized that at some point you got the two boxes mixed up, and you started putting the dead batteries in with the new batteries.
Each box now has 20 batteries. The "good" box has 15 good batteries and 5 dead batteries. The "bad" box has 15 dead batteries and 5 good batteries.
You pick a box at random and test two batteries. Both of the batteries are dead.
What is the probability that you have choosen the bad box?

B = Bad Box

G = Good Box

D = Both dead

$$P(B|D) = \frac{P(D|B)\cdot P(B)}{P(D)} = \frac{\frac{\binom{15}{2}}{\binom{20}{2}}\cdot\frac{1}{2}}{0.3026} = 0.91304$$

But $P(D) = P(D|B)P(B) + P(D|G)\cdot P(G)$

$$= \frac{\binom{15}{2}}{\binom{20}{2}}\cdot\frac{1}{2} + \frac{\binom{5}{2}}{\binom{20}{2}}\cdot\frac{1}{2} = 0.3026$$

# 7.6: Baye's Theorem

- A and B are events in a probability space, so that neither $P(A) = 0$ nor $P(B) = 0$.

- Then

$$P(A|B) = \frac{P(B|A)P(A)}{P(B)}$$

# 7.6: Flip you for it?

$A =$ 58% Heads coin
$B =$ 30% Heads coin
$C =$ Flip 3 times & 2 heads
$H =$ Heads on next flip

- Your friend has a biased coin, which has a 58% of turning up heads. He has another biased coin, with 30% chance of heads.
- Your friend picks one of the coins at random. He flips it three times. It comes up heads twice.
- He offers you the following deal: He'll buy you coffee every morning for the next year if the coin comes up tails, and you'll have to clean his apartment every weekend for the next year if the coin comes up heads.
- What is the probability you'll be cleaning his apartment?

Want $P(H|C)$.

By definition, $P(H|C) = \dfrac{P(H \cap C)}{P(C)}$

But $A$ & $B$ are mutually exclusive & Exhaustive,

So $P(H \cap C) = P(H \cap C|A) \cdot P(A) + P(H \cap C|B) \cdot P(B)$

& $P(C) = P(C|A)P(A) + P(C|B)P(B)$

Also, $P(A) = P(B) = \frac{1}{2}$

$$P(C|A) = \binom{3}{2} (.58)^2 (.42)$$

$$P(C|B) = \binom{3}{2} (.3)^2 (.7)$$

So $P(C) = \binom{3}{2} (.58)^2 (.42) \cdot \frac{1}{2} + \binom{3}{2} (.3)^2 (.7)$

Next, $P(H \cap C | A)$?

Note that, conditioned on $A$, $H$ & $C$ are independent

$$P(H \cap C | A) = P(H|A) \cdot P(C|A)$$

$$= (.58) \cdot \binom{3}{2} (.58)^2 \cdot (.42)$$

Also, $H$ & $C$ are independent conditional on $B$

$$P(H \cap C | B) = P(H|B) \cdot P(C|B)$$

$$= (.3) \binom{3}{2} \cdot (.3)^2 (.7)$$

So, $P(H|C) = \dfrac{P(H \cap C|A) P(A) + P(H \cap C|B) P(B)}{P(C|A) P(A) + P(C|B) P(B)}$

$$= \frac{(.58) \cdot \binom{3}{2} \cdot (.58)^2 (.42) \cdot \frac{1}{2} + (.3) \binom{3}{2} (.3)^2 (.7) \cdot \frac{1}{2}}{\binom{3}{2} (.58)^2 (.42) \cdot \frac{1}{2} + \binom{3}{2} (.3)^2 (.7) \cdot \frac{1}{2}}$$

$$= 0.49365$$

You still have a slighter better chance of coming out ahead.

You'll clean his apartment w/ Prob 49.365%

& You'll get free coffee w/ Prob 50.635%