MA162: Finite mathematics

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Schedule:

- Web Assign assignment (Chapter 3.1) due on Tuesday, October 8 by 6:00 pm.
- Web Assign assignment (Chapter 3.2) due on Friday, October 11 by 6:00 pm.
- Web Assign assignment (Chapter 3.3) due on Tuesday, October 15 by 6:00 pm.

Today we cover Chapter 3.2: Linear Programming Problems.

Linear Programming Problems

A *linear programming problem* consists of

- a linear *objective function*
- a collection of *constraints*, each in the form of a linear equality or linear equality.

The goal of a linear programming problem is to maximize or minimize the objective function, while satisfying all of the constraints.

A very simple linear programming problem

- A farmer has 100 acres of land.
- The farmer can use the land to grow corn or wheat.
- For each acre of corn, the farmer earns $651.
- For each acre of wheat, the farmer earns $523.
- In order to maximize his revenue, how many acres should be used for corn, and how many acres for wheat.

The solution

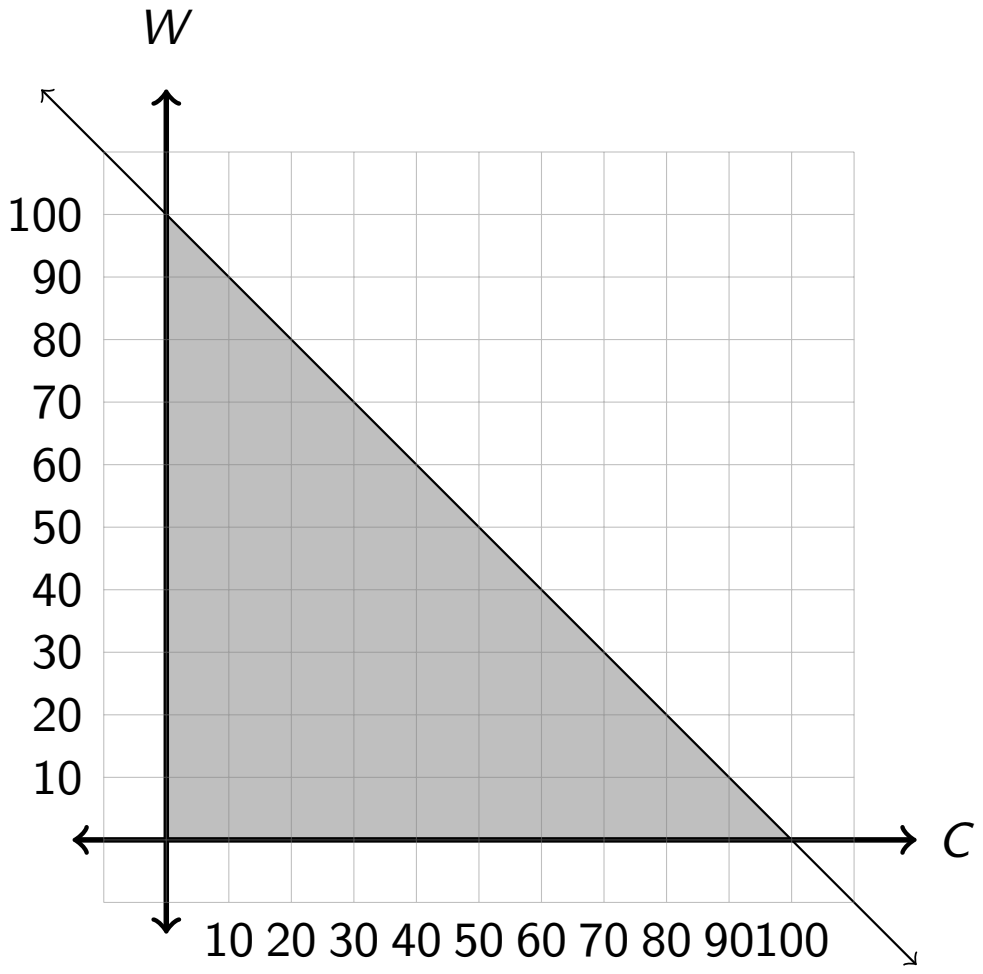
We can find the answer to *this* problem without using any fancy techniques.

- The farmer earns more from corn than from wheat, so farmer should devote all available land to corn.
- Thus, 100 acres, all devoted to corn, \$651 per acre, so maximum revenue is $\$65,100$
- Not all LPs are this straightforward, so lets look at a more robust method.

Setting up the simple linear programming problem

- Let C denote number of acres of corn, W the number of acres of wheat.
- The **objective function** is $R = 651 \cdot C + 523 \cdot W$.
- There are three **constraints**:
  - $C \geq 0$
  - $W \geq 0$
  - $C + W \leq 100$

Solving the linear programming problem



A less simple linear programming problem

- A farmer has 100 acres of land to grow corn or wheat.
- Farmer earns \$651 for each acre of corn and \$523 for each acre of wheat.
- Harvesting the corn requires 20 hours of labor per acre.
- Harvesting the wheat requires 12 hours of labor per acre.
- The farmer has enough workers for 1500 hours of labor.
- In order to maximize his revenue, how many acres should be used for corn, and how many acres for wheat.

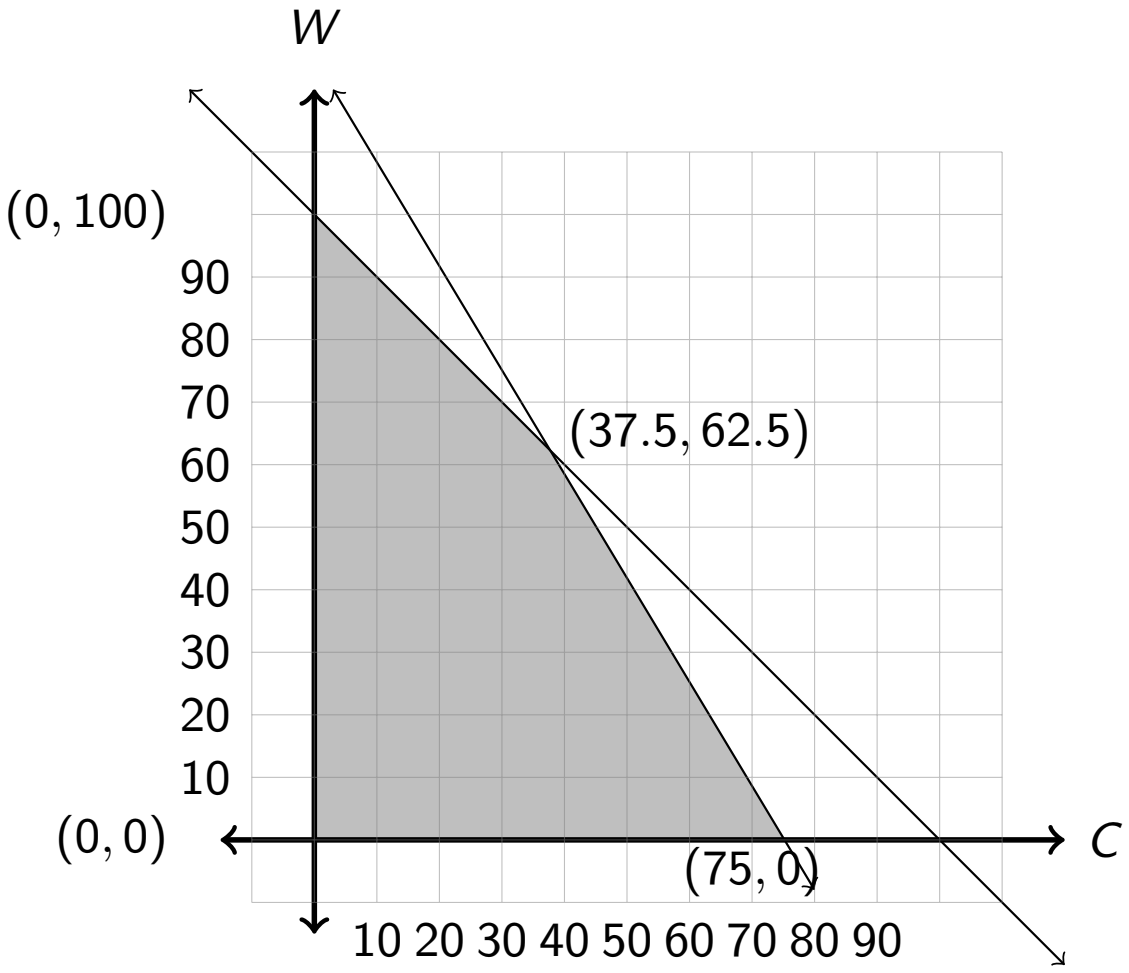
Solution?

The “use all 100 acres for corn” is no longer a valid solution, as this would require 2000 hours of labor, but the farmer only has 1500 hours available.

Setting up the linear programming problem

- Let C denote number of acres of corn, W the number of acres of wheat.
- The **objective function** is $R = 651 \cdot C + 523 \cdot W$.
- There are four **constraints**:
  - $C \geq 0$
  - $W \geq 0$
  - $C + W \leq 100$
  - $20 \cdot C + 12 \cdot W \leq 1500$

Solving the linear programming problem



The Method of Corners

- Graph the feasible set.
- Find the coordinates of all of the corner points of the feasible set.
- Evaluate the objective function at each corner.
- Theorems 1 and 2 from the text guarantee that the objective function reaches a maximum at one of these corner points, and a minimum at another corner point, provided the feasible set is bounded.

A Nutrition Example

- A Food-and-Nutrition-Science student was asked to design a diet for someone with iron and vitamin B deficiencies
- The student said the person should get at least 2400mg of iron, 2100mg of vitamin B_1, and 1500mg of vitamin B_2 (over 90 days)
- The student recommended two brands of vitamins:

	Brand A	Brand B	Min. Req
Iron	40mg	10mg	2400mg
B_1	10mg	15mg	2100mg
B_2	5mg	15mg	1500mg
Cost:	\$0.06	\$0.08	

- The client asked the student to recommend the cheapest solution
- How many pills of each brand should the person get in order to meet the nutritional requirements at the minimal cost?

Shipping costs example

- You hit the big time, Mr. or Ms. Big Shot.
  You've got two manufacturing plants and two assembly plants
- Your assembly plants A1 and A2 need 80 and 70 engines
- Your production plants can produce up to 100 and 110 engines
- The shipping costs are:

	To assembly plant	
From	A1	A2
P1	100	60
P2	120	70

- How many engines should each production plant ship to each assembly plant to meet the production goals at the minimum shipping cost?