

MA162: Finite mathematics

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October 9, 2013

SCHEDULE:

- Web Assign assignment (Chapter 3.3) due on Tuesday, October 15 by 6:00 pm.
- Web Assign assignment (Chapter 4.1) due on Friday, October 18 by 6:00 pm.
- Exam 2 on Monday, October 28, 5:00 pm to 7:00 pm.

Today we cover Chapter 3.3: Graphical Solution to Linear Programming Problems.

Graphical Solution to LPs

Recall that a linear programming problem consists of

- an **objective function** to be maximized or minimized
- a collection of **constraints**, in the form of linear equations and linear inequalities.

Terminology

- The set of all solutions to the constraints is called the **feasible set**.
- Any point in the feasible set is called a **feasible solution**.
- Any point outside of the feasible set is called **infeasible**.
- Feasible/infeasible are concerned with which points are allowed. They tell us nothing about the **optimal point**

Theorem 1

If a Linear Programming Problem has a solution, then the solution must occur at a corner point of the feasible set.

Furthermore, if the objective function is optimized at two adjacent corners, then the objective function is also optimized at any point on the line segment joining these adjacent corners. In this case, the LPP has infinitely many solutions.

Theorem 2

Feasible set is S , objective function is $P = ax + by$

- If S is *bounded* then P has both a maximum value and a minimum value on S .
- If S is unbounded and a and b are both nonnegative, then P has a minimum value provided that the constraints defining S include $x \geq 0$ and $y \geq 0$.
- If S is the empty set then the LPP has no solution. P has neither a maximum nor a minimum.

Method of Corners

- To solve an LPP, first use Theorem 2 to decide if the LPP has a solution.
- Graph the feasible set
- Evaluate the objective function at each corner point
- The optimal solution point which produces the largest (or smallest) value found in the above step.

A Nutrition Example

- A Food-and-Nutrition-Science student was asked to design a diet for someone with iron and vitamin B deficiencies
- The student said the person should get at least 2400mg of iron, 2100mg of vitamin B_1 , and 1500mg of vitamin B_2 (over 90 days)
- The student recommended two brands of vitamins:

	Brand A	Brand B	Min. Req
Iron	40mg	10mg	2400mg
B_1	10mg	15mg	2100mg
B_2	5mg	15mg	1500mg
Cost:	\$0.06	\$0.08	

- The client asked the student to recommend the **cheapest** solution
- How many pills of each brand should the person get in order to meet the nutritional requirements at the minimal cost?

Shipping costs example

- You hit the big time, Mr. or Ms. Big Shot.
You've got two manufacturing plants and two assembly plants
- Your assembly plants A1 and A2 need 80 and 70 engines
- Your production plants can produce up to 100 and 110 engines
- The shipping costs are:

From	To assembly plant	
	A1	A2
P1	100	60
P2	120	70

- How many engines should each production plant ship to each assembly plant to meet the production goals at the minimum shipping cost?

Tan, Chapter 3.2 Exercise 5

Winston Furniture Company manufactures tables and chairs. Each table requires 40 board feet of wood and 3 labor-hours. Each chair requires 16 board feet of wood and 4 labor-hours. Profit for each table is \$45 and profit for each chair is \$20. In a certain week, the company has 3200 board feet available and 520 labor-hours available. How many tables and how many chairs should they produce to maximize their profit?