MA162: Finite mathematics

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SCHEDULE:

- Web Assign assignment (Chapter 4.1) due on Friday, October 18 by 6:00 pm.
- Web Assign assignment (Chapter 4.1) due on Tuesday, October 22 by 6:00 pm.
- Exam 2 on Monday, October 28, 5:00 pm to 7:00 pm.

Today we continue our discussion on the Simplex Algorithm, Chapter 4.1.

Exercise 17, Chapter 4.1

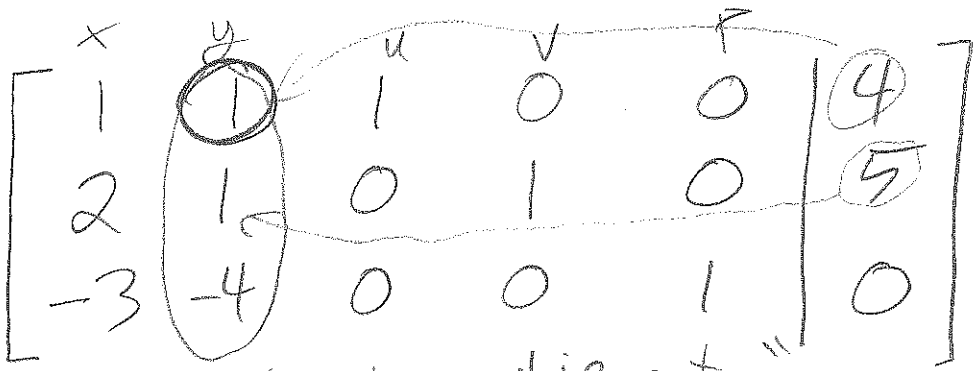
Use the simplex algorithm to solve this LPP:

Maximize: $P = 3x + 4y$ $\Longrightarrow$ $-3x - 4y + P = 0$

Constraints:

Add in slack variables.

- $x + y \leq 4$ $\Longrightarrow$ $x + y + u = 4$
- $2x + y \leq 5$ $\Longrightarrow$ $2x + y + v = 5$
- $x \geq 0$, ~~≥ 0~~ $y \geq 0$, $u \geq 0$, $v \geq 0$



$4/1 = 4$ ← Smallest. So use Row 1 to pivot

$5/1 = 5$

$R_2 \mapsto R_2 - 2R_1$

$R_3 \mapsto R_3 + 4R_1$

Pivot row 1, column 2

x	y	u	v	P	
1	1	1	0	0	4
1	0	-1	1	0	1
1	0	4	0	1	16

← No more negative entries in bottom row, so done.

Now interpret:

y, v, P are pivot variables,

x + u are slack,

Set slack to zero,

Get $x = 0$, $y = 4$, $u = 0$, $v = 1$, $P = 16$.

Exercise 25, Chapter 4.1

Use the simplex algorithm to solve this LPP:

Maximize: $P = 3x + 4y + z$ $\Rightarrow$ $-3x - 4y - z + P = 0$

Constraints:

- $3x + 10y + 5z \leq 120$ $\quad 3x + 10y + 5z + u = 120$
- $5x + 2y + 8z \leq 6$ $\quad 5x + 2y + 8z + v = 6$
- $8x + 10y + 3z \leq 105$ $\quad 8x + 10y + 3z + w = 105$
- $x \geq 0, y \geq 0, z \geq 0$ + $u \geq 0, v \geq 0, w \geq 0$

$$\begin{bmatrix} 3 & 10 & 5 & 1 & 0 & 0 & 0 & 120 \\ 5 & 2 & 8 & 0 & 1 & 0 & 0 & 6 \\ 8 & 10 & 3 & 0 & 0 & 1 & 0 & 105 \\ -3 & -4 & -1 & 0 & 0 & 0 & 1 & 0 \end{bmatrix}$$

$120/10 = 12$

$6/2 = 3$ ← Smallest ratio

$105/10 = 10.5$

↑ Most negative in bottom row

$R_2/2 \rightarrow$

$$\begin{bmatrix} 3 & 10 & 5 & 1 & 0 & 0 & 0 & 120 \\ 5/2 & 1 & 4 & 0 & 1/2 & 0 & 0 & 3 \\ 8 & 10 & 3 & 0 & 0 & 1 & 0 & 105 \\ -3 & -4 & -1 & 0 & 0 & 0 & 1 & 0 \end{bmatrix}$$

$$\begin{matrix} & x & y & z & u & v & w & p \end{matrix}$$

$R_1 - 10R_2$

$R_3 - 10R_2$

$R_4 + 4R_2$

$$\begin{bmatrix} -22 & 0 & -35 & 1 & -5 & 0 & 0 & 90 \\ 2.5 & 1 & 4 & 0 & 0.5 & 0 & 0 & 3 \\ -17 & 0 & -37 & 0 & -5 & 1 & 0 & 75 \\ 7 & 0 & 15 & 0 & 2 & 0 & 1 & 12 \end{bmatrix}$$

← No more negative entries in bottom row

So we just need to interpret answer

Set slack variables to 0, so

$x = 0, \quad z = 0, \quad v = 0$

Then $u = 90$

$y = 3$

$w = 75$

$P = 12$

Exercise 46, Chapter 4.1

Boise Lumber manufactures prefabricated houses. They offer three models, standard, deluxe, and luxury.

Each house is prefabricated and partially assembled in a factory. The final assembly is done on site.

The dollar amount of building material required, the amount of factory labor required, and the amount of on-site labor required, as well as profit per unit are

	Standard	Deluxe	Luxary
Material	6000	8000	10000
Factory labor	240	220	200
On-site labor	180	210	300
Profit	3400	4000	5000

They have $\$8,200,000$ budgeted for building materials, $218,000$ hours for factory labor, and $237,000$ labor hours for on-site labor.

How many houses of each type should they build in order to maximize their profit?

Let x = # standard, y = # deluxe, z = # luxury.

Then
$$6000x + 8000y + 10000z \le 8{,}200{,}000 \quad \text{Material}$$
$$240x + 220y + 200z \le 218{,}000 \quad \text{Fact. labor}$$
$$180x + 210y + 300z \le 237{,}000 \quad \text{on-site labor}$$
$$P = 3400x + 4000y + 5000z \quad \text{Profit}$$
$$x, y, z \ge 0$$

Let u = unused material,
v = unused fact. labor
w = unused on-site labor

Then
$$\begin{cases} 6000x + 8000y + 10000z + u = 8200000 \\ 240x + 220y + 200z + v = 218000 \\ 180x + 210y + 300z + w = 237000 \\ -3400x - 4000y - 5000z + P = 0 \\ x, y, z, u, v, w \ge 0 \end{cases}$$

Simplex Tableau

$$\left[\begin{array}{cccccccc|c} 6000 & 8000 & 10000 & 1 & 0 & 0 & 0 & 8200000 \\ 240 & 220 & 200 & 0 & 1 & 0 & 0 & 218000 \\ 180 & 210 & 300 & 0 & 0 & 1 & 0 & 237000 \\ -3400 & -4000 & \textcircled{-5000} & 0 & 0 & 0 & 1 & 0 \end{array}\right]$$

Column 3 to pivot

Now look at ratios Right hand side to Column 3:

$\frac{8200000}{10000} = 820$

$\frac{218000}{200} = 1090$

$\frac{237000}{300} = 790$ Use row 3 to pivot

$R_3/300 \Rightarrow$

$$\left[\begin{array}{ccccccc|c} 6000 & 8000 & 10000 & 1 & 0 & 0 & 0 & 8200000 \\ 240 & 220 & 200 & 0 & 1 & 0 & 0 & 218000 \\ 0.6 & 0.7 & 1 & 0 & 0 & 1/3 & 0 & 790 \\ -3400 & -4000 & -5000 & 0 & 0 & 0 & 1 & 0 \end{array}\right]$$

$R_1 - 10000R_3$
$R_2 - 200R_3$
$R_4 + 5000R_3$

$$\left[\begin{array}{ccccccc|c} 0 & 1000 & 0 & 1 & 0 & -33.33 & 0 & 300000 \\ 120 & 80 & 0 & 0 & 1 & -2/3 & 0 & 60000 \\ 0.6 & 0.7 & 1 & 0 & 0 & 1/300 & 0 & 790 \\ -400 & -500 & 0 & 0 & 0 & 16.67 & 1 & 3950000 \end{array}\right]$$

↑ Most negative, so use this column to pivot

Ratios: $300000/1000 = 300$ ← Use row 1 to pivot

$60000/80 = 750$

$790/0.7 = 1128.57$

$R_1/1000 \rightarrow$

$$\left[\begin{array}{cccccccc|c} 0 & 1 & 0 & 0.001 & 0 & -0.0\overline{3} & 0 & 300 \\ 120 & 80 & 0 & 0 & 1 & -0.\overline{6} & 0 & 60000 \\ 0.6 & 0.7 & 1 & 0 & 0 & 0.00\overline{3} & 0 & 790 \\ -400 & -500 & 0 & 0 & 0 & 16.\overline{6} & 1 & 3950000 \end{array}\right]$$

$R_2 - 80R_1$
$R_3 - 0.7R_1$
$R_4 + 500R_1$

$$\left[\begin{array}{cccccccc|c} 0 & 1 & 0 & 0.001 & 0 & -0.0\overline{3} & 0 & 300 \\ 120 & 0 & 0 & -0.08 & 1 & 2 & 0 & 36000 \\ 0.6 & 0 & 1 & -0.0007 & 0 & 0.02\overline{6} & 0 & 580 \\ -400 & 0 & 0 & 0.5 & 0 & 0 & 1 & 4{,}100{,}000 \end{array}\right]$$

↑ Now pivot on first column

Ratios:

$300/0 \leftarrow$ Don't

$\frac{36000}{120} = 300 \leftarrow$ Second row to pivot.

$\frac{580}{0.6} = 966.\overline{6}$

$R_2/120$

$$\left[\begin{array}{cccccccc|c} 0 & 1 & 0 & 0.001 & 0 & -0.0\overline{3} & 0 & 300 \\ 1 & 0 & 0 & -0.000\overline{6} & 0.00\overline{83} & 0.0\overline{16} & 0 & 300 \\ 0.6 & 0 & 1 & -0.0007 & 0 & 0.02\overline{6} & 0 & 580 \\ -400 & 0 & 0 & 0.5 & 0 & 0 & 1 & 4{,}100{,}000 \end{array}\right]$$

$R_3 - 0.6R_2$, $R_4 + 400R_2$

x	y	z	u	v	w	P	
0	1	0	0.001	0	$-0.0\overline{3}$	0	300
1	0	0	$-0.000\overline{6}$	$0.00\overline{83}$	$0.0\overline{16}$	0	300
0	0	1	-0.0003	-0.005	$0.0\overline{16}$	0	400
0	0	0	$0.23\overline{3}$	$3.\overline{3}$	$6.\overline{6}$	1	4220000

All entries in row 4 are non-negative, so we are done.

FINALLY

x, y, z, P are pivot variables, u, v, w are slack.

Set slack to zero.

Then get $y = 300$, $x = 300$, $z = 400$, $P = 4,220,000$.

Interpretation:

Should make 300 standard, 300 deluxe, 400 luxury.

Will then earn 4,220,000 in profit.

$u = 0 \Rightarrow$ No unused material

$v = 0 \Rightarrow$ No unused factory labor

$w = 0 \Rightarrow$ No unused on-site labor.