MA162: Finite mathematics

Paul Koester

University of Kentucky

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SCHEDULE:

- Web Assign assignment (Chapter 6.1 and 6.2) due on Friday, October 25 by 6:00 pm.
- Exam 2 on Monday, October 28, 5:00 pm to 7:00 pm.

Today we look at Chapter 6.2

6.2: A two set inclusion-exclusion

- A survey of 5000 subscribers to a certain news paper revealed that 2700 people subscribe to the daily morning edition and 1800 subscribe to both the daily and the Sunday editions.

D = Subscibe to Daily, S = subscribe to Sunday

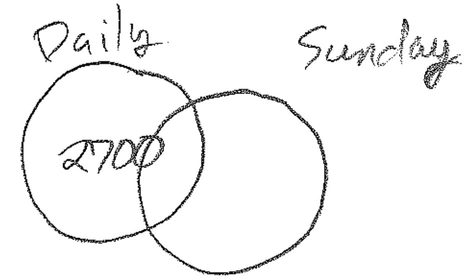
$n(D) = 2700$, $n(S \cap D) = 1800$.

- How many subscribe to the Sunday edition?

$n(S \cup D) = 5000$.

So $n(S \cup D) = n(S) + n(D) - n(S \cap D)$

$5000 = n(S) + 2700 - 1800 \Longrightarrow n(S) = 4100$



- How many subscribe to the Sunday edition only?

$n(S \cup D) - n(D)$

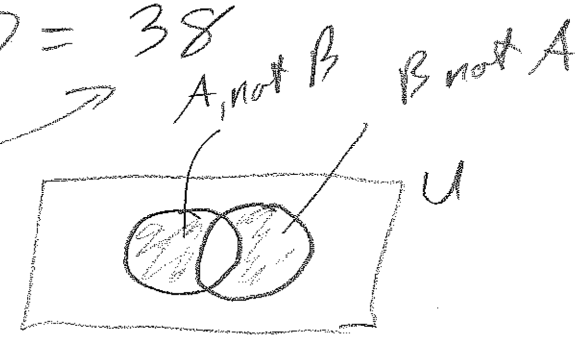
$= 5000 - 2700 = 2300$.

6.2: Inclusion-exclusion with diagrams

- Given that $n(A \cap B) = 7$, $n(A) = 23$, $n(B) = 18$, and $n(U) = 45$, find each of the following:

It should have asked $n(A \cup B)$,

- $n(A \cap B) = 7,$ $\quad n(A \cup B) = n(A) + n(B) - n(A \cap B)$
  $= 23 + 18 - 7 = 34$
- $n(A^c \cap B^c) = n((A \cup B)^c) = n(U) - n(A \cup B) = 45 - 34 = 11$
- $n[(A \cap B)^c] = n(U) - n(A \cap B) = 45 - 7 = 38$
- $n(A^c \cup B^c) = n((A \cap B)^c) = 38$



A & not B OR B & not A

- $n[(A \cap B^c) \cup (A^c \cap B)] = n(A \cup B) - n(A \cap B) = 34 - 7 = 27$
- $n(U^c) = 0$

6.2: Another two set inclusion-exclusion

$n(D) = 400, \quad n(C) = 290, \quad n(D \cup C) = 500$

$n(D \cap C) = 400 + 290 - 500 = 190$

- Of 500 clock radios with digital tuners and/or CD players sold recently in a department store, 400 had digital tuners and 290 had CD players.

All 500 radios have one or the other, so same as $n(D^c)$

- How many radios had a CD player but not a digital tuner?

$n(C - D) = n(C \cap D^c) = n(D^c) = 500 - 400 = 100$

- How many radios had a CD player or a digital tuner, but not both?

$n((C \cap D^c) \cup (D \cap C^c)) = n(C \cup D) - n(C \cap D)$

$= 500 - 190 = 310$

6.2: A three set inclusion-exclusion

- Let A, B, and C be subsets of a universal set U and suppose $n(U) = 200$, $n(A) = 21$, $n(B) = 23$, $n(C) = 27$, $n(A \cap B) = 8$, $n(A \cap C) = 12$, $n(B \cap C) = 15$, and $n(A \cap B \cap C) = 3$. Compute:

$n(A \cup B \cup C) = 21 + 23 + 27 - 8 - 12 - 15 + 3 = 39$

$n(A \cap (B \cup C)) = n(A \cap B) + n(A \cap C) - n(A \cap B \cap C)$

- Find $\cancel{n}[A\cancel{(B \cup C)}]$ $= 8 + 12 - 3 = 17$

Note: $n(A) = n(A \cap (B \cup C)) + n(A \cap (B \cup C)^c)$

- Find $\cancel{n}[A\cancel{(B \cup C)}^c]$

$n(A \cap (B \cup C)^c) = n(A) - n(A \cap (B \cup C))$
$= 21 - 17 = 4$

6.2: Another three set inclusion-exclusion

A survey of a group's viewing habits over the last year revealed the following information:

- 28% watched gymnastics
- 29% watched baseball
- 19% watched soccer
- 14% watched gymnastics and baseball
- 12% watched baseball and soccer
- 10% watched gymnastics and soccer
- 8% watched all three sports

Calculate the percentage of the group that watched none of the three sports during the last year.

$n(G) = 28\%$

$n(B) = 29\%$

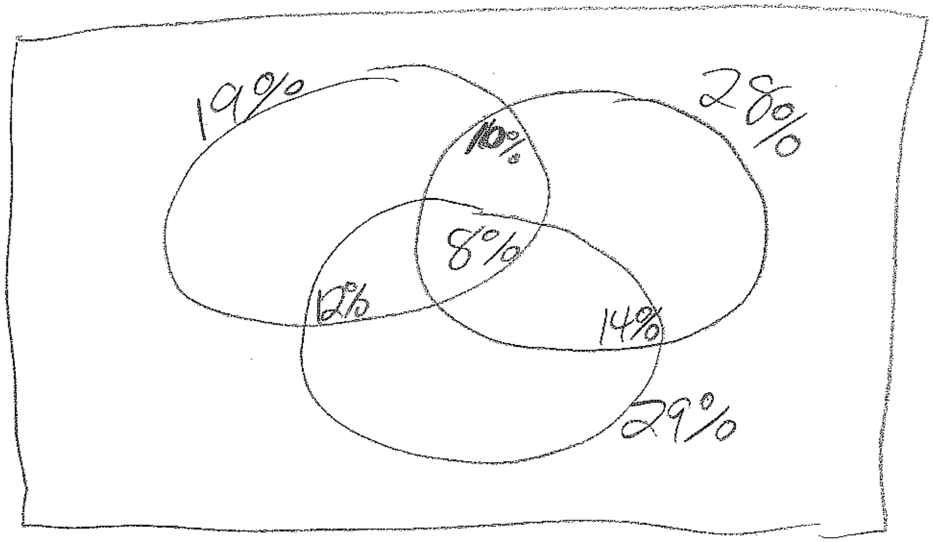
$n(S) = 19\%$

$n(G \cap B) = 14\%$

$n(B \cap S) = 12\%$

$n(G \cap S) = 10\%$

$n(G \cap B \cap S) = 8\%$



$$n(G \cup B \cup S) = n(G) + n(B) + n(S) - n(G \cap B) - n(G \cap S) - n(B \cap S) + n(G \cap B \cap S)$$

$$= 28 + 29 + 19 - 14 - 12 - 10 + 8 = 48\%$$

48% watched at least one,

so 52% watched none.

6.2: Another inclusion-exclusion

An insurance company examines its pool of 1000 auto insurance customers and gathers the following information:

- All customers insure at least one car.
- 640 of the customers insure more than one car.
- 200 of the customers insure a sports car.
- Of those customers who insure more than one car, 15% insure a sports car.

How many customers insure exactly one car, and that car is not a sports car?

One = { own one Car }, One^c = { own more than one car }

S = { own sports Car }

$n(One) = 640$, $n(One^c) = 1000 - 640 = 360$

$n(S) = 200$, $n(One \cup One^c) = 1000$

$n(One^c \cap S) = 15\%$ of $n(One^c)$

$= 0.15 \cdot 360 = 54$

Want: $n(One \cap S^c)$

$= 496$

	One Car	More than one	
Sports Car	$200 - 54$ $= 146$	54	200 own sports
No Sports Car	$640 - 146$ $= 496$		800 don't own sports
	640 insure one car	360 insure > one	