MA162: Finite mathematics

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SCHEDULE:

- First Web Assign assignment due on Friday, September 6 by 6:00 pm.
- Second Web Assign assignment due on Tuesday, Septemeber 10 by 6:00 pm.
- "Miniature exam" over Chapter 1 will be given in recitation on either Tuesday, September 10 or Thursday, September 12.

Today we cover chapters 1.3 and 1.4. These sections cover linear models and intersections of lines. These sections introduce several important applications, like *depreciation*, *cost-revenue analysis*, and *supply-demand equilibrium analysis.*

Tan, Chapter 1.2, Exercise 72 I

For wages less than the maximum taxable wage base, Social Security contributions by employees are 7.65% of the employee's wages.

(a.) Find an equation that expresses the relationship between wages earned (x) and Social Security taxes paid (y) by an employee who earns less than the maximum taxable wage base.

(b.) For each additional dollar that an employee earns, by how much is his or her Social Security contribution increased?

(c.) What Social Security contributions will and employee who earns $\$65,000$ be required to make?

Tan, Chapter 1.2, Exercise 72, alternative wording I

For wages less than the maximum taxable wage base, Social Security contributions by employees are 7.65% of the employee's wages. Let x denote the wages earned and let y denote the Social Security taxes paid by employee who earns less than the maximimum taxable wage base. Thus,

$$y = 0.0765 \cdot x$$

You earn $\$65,000$ and you want to determine your Social Security contribution. (NOTE: $\$65,000$ is below the maximum taxable wage rate, so the above formula applies to this employee.) You should

(a.) Set $x = 65000$ and solve for y.

(b.) Set $y = 65000$ and solve for x.

(c.) Not pay anything and hope the government doesn't catch you.

Cost, Revenue, Profit, Break-even I

You have 100 acre plot of land. You plan to grow and sell corn.

- You will need to buy a plow, which costs $\$17,000$.
- Costs \$340 per acre for seed, fertilizer, and pesticides.
- Corn currently sells for \$4.82 per bushel. (Or, look up current price here: http://www.quotecorn.com/)
- One acre of corn yields 135 bushels of corn.
- For simplicity, we will ignore all other costs (rent on land, taxes, labor, fuel, storage, shipping, etc.)

The Cost Function I

(a.) How much does it cost to grow 20 acres of corn?

(b.) How much does it cost to grow 70 acres of corn?

(c.) How much does it cost to grow x acres of corn? (Call this $C(x)$.)

(d.) In plain English, what does the slope mean?

(e.) In plain English, what does the vertical intercept mean?

The Revenue Function I

(a.) How much revenue do you generate by selling 20 acres of corn?

(b.) How much revenue do you generate by selling 70 acres of corn?

(c.) How much revenue do you generate by selling x acres of corn?

(d.) In plain English, what does the slope mean?

(e.) In plain English, what does the vertical intercept mean?

Cost, Revenue, Profit I

(a.) If you grow and sell 20 acres of corn, which is greater, the cost or the revenue?

(b.) If you grow and sell 70 acres of corn, which is greater, the cost or the revenue?

(c.) Determine your profit earned from growing and selling x acres of corn. (Call this $P(x)$.)

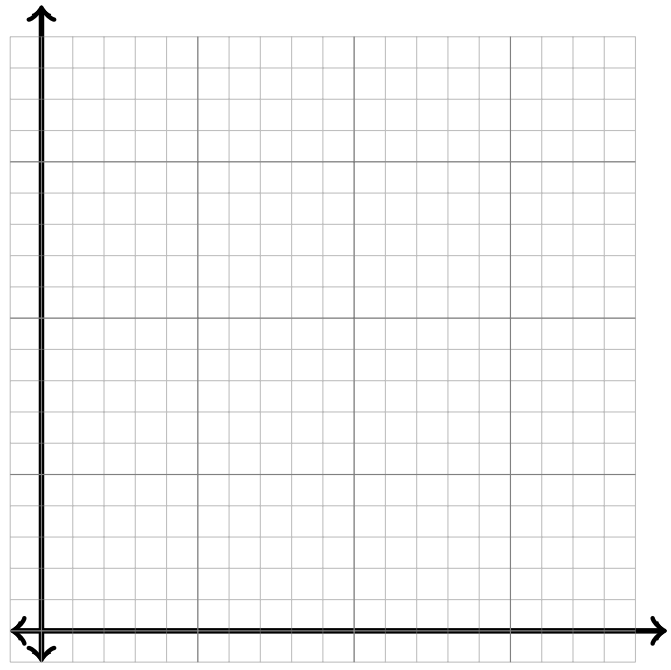
(d.) In plain English, what does the slope mean?

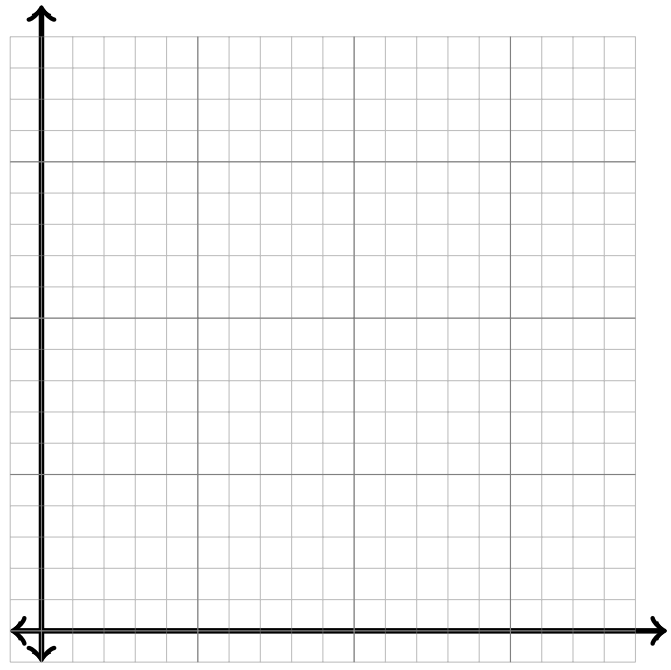
(e.) In plain English, what does the vertical intercept mean?

Profit and Break-Even I

(a.) Determine the value of x at which $P(x) = 0$. (This is equivalent to finding x where $C(x) = R(x)$.

(b.) In plain English, what does this x-intercept mean?





Linear Depreciation I

You purchased a new car in 2010. You paid $\$20,000$ for the car. You estimate that you can drive the car for 10 years before it will break down and need to be replaced. You also estimate that you will be able to sell the car to a junk yard for $\$1,000$ when it breaks down.

(a.) What is the value of the car in 2012?

(b.) What is the value of the car in 2018?

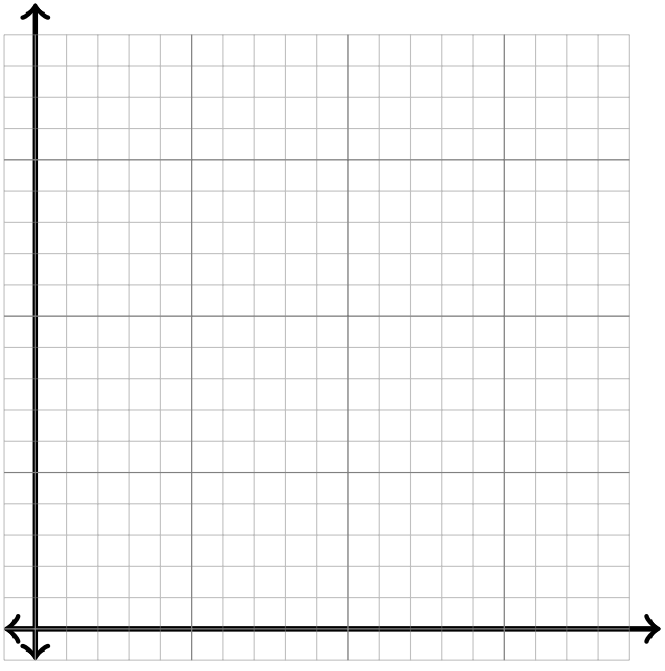
(c.) Let $V(t)$ denote the value of the car (in dollars), where t is the age of the car in years. Find $V(t)$.

(d.) In plain English, what does the slope of $V(t)$ mean?

Linear Depreciation II

(e.) In plain English, what does the vertical intercept mean?

(f.) Sketch the graph of $V(t)$.



Supply, Demand, and Equilibrium

- Your tech company is about to unveil its new *Me-Pod*.
- Your task is to determine how much to charge for this device.
- Charge too little, then your company will sell a lot of these devices, but will generate very little revenue on each individual sale.
- Charge too much, then your company will generate a lot of revenue on each individual sale, but very few of these devices will be sold.
- What to do?

The Demand Function I

First, you ask your marketing team to determine how many of these devices your company might expect to sell at various prices. Their results:

- Expect to sell 1000 units per month if you charge \$200 per Me-Pod.
- Expect to sell 800 units per month if you charge \$250 per Me-Pod.

Let x denote the number of Me-Pods sold per month and let p denote the price per Me-Pod.

(a.) Express p as a function of x, assuming the variables are linearly related.

(b.) Determine the slope, the vertical intercept, and the horizontal intercept.

The Demand Function II

(c.) In plain English, explain what the slope means.

(d.) In plain English, explain what the vertical-intercept means.

(e.) In plain English, explain what the horizontal-intercept means.

The Supply Function I

Next, you ask your manufacturing division to determine how many of these devices they are willing to produce at various prices. Their answer:

- Willing to produce 600 units per month if they sell for \$200 per Me-Pod.
- Willing to produce 1000 units per month if they sell for \$250 per Me-Pod.

Let x denote the number of Me-Pods sold per month and let p denote the price per Me-Pod.

(a.) Express p as a function of x, assuming the variables are linearly related.

(b.) Determine the slope, the vertical intercept, and the horizontal intercept.

The Supply Function II

(c.) In plain English, explain what the slope means.

(d.) In plain English, explain what the vertical-intercept means.

(e.) In plain English, explain what the horizontal-intercept means.

Equilibrium I

So what is the "correct" price?
The equilibrium quantity and equilibrium price correspond to the point where the supply curve and the demand curve intersect.

(a.) Determine the equilibrium price and determine the equilibrium quantity.

(b.) In plain English, explain what the equilibrium quantity means.

(c.) In plain English, explain what the equilibrium price means.