

MA162: Finite mathematics

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SCHEDULE:

- Fourth Web Assign assignment (Chapter 2.2) due on Tuesday, September 17 by 6:00 pm.
- Fifth Web Assign assignment (Chapter 2.3) due on Friday, September 20 by 6:00 pm.
- Exam 1 on Monday, September 30 from 5:00 pm to 7:00 pm

Today we cover chapter 2.3. This section continues developing the Gauss-Jordan Method. Specifically, we'll focus on cases where there is either no solution and when there are infinitely many solutions.

An Overdetermined System

Find the solution to the system of equations

$$\begin{array}{rclclcl} x & + & y & + & z & = & 1 \\ 4x & + & 3y & + & 5z & = & 7 \\ 2x & + & y & + & 3z & = & 6 \end{array}$$

An Underdetermined System

Find the solution to the system of equations

$$4x_1 + 5x_2 + 6x_3 = 7$$

$$x_1 + 2x_2 + 3x_3 = 4$$

$$7x_1 + 8x_2 + 9x_3 = 10$$

Identifying Systems with No Solution

- Its not easy to determine if system has no solution or infinitely many solutions just by looking at augmented matrix.
- Start doing Gauss-Jordan.
- At any point, if you see a row in which all entries on the left side of the vertical bar are zero, and the entry to the right of the bar is nonzero, then the system has NO SOLUTION.

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$$\left[\begin{array}{ccc|c} 1 & 3 & -2 & 7 \\ 0 & 4 & 1 & -3 \\ 0 & 0 & 0 & 2 \end{array} \right] \leftarrow \text{No Solution!}$$

The associated system of equations has no solution.

- Don't necessarily need to go all the way to RREF!

Identifying Systems with Infinitely Many Solutions

- Put the augmented matrix into row-reduced echelon form.
- Check that there are no rows in which all entries on the left are zero and the entry on the right is nonzero.
- Is the number of nonzero rows less than the number of columns on the left side of the vertical bar?
 - More columns than nonzero rows implies infinitely many solutions.
 - Same number of nonzero rows as columns implies exactly one solution.
- This test can give incorrect results if the augmented matrix has not been put in RREF!

Identifying Systems with Infinitely Many Solutions, cont.

- $$\left[\begin{array}{ccc|c} 1 & 2 & 0 & 7 \\ 0 & 0 & 1 & -3 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array} \right]$$

2 nonzero rows, 3 columns on left side of vertical bar, so this system has infinitely many solutions.

- $$\left[\begin{array}{ccc|c} 1 & 2 & 0 & 7 \\ 0 & 0 & 0 & -3 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array} \right]$$

Row with all zeros on left side of vertical bar, nonzero on right side of vertical bar, NO SOLUTION

Identifying Systems with Infinitely Many Solutions, cont.



$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 7 \\ 0 & 1 & 0 & -3 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{array} \right]$$

3 nonzero rows, 3 columns on left side of vertical bar, so this system has exactly one solution.

Another Underdetermined System

Find the solution to the system of equations

$$4x + 5y + 3z + 3w = 1$$

$$x + y + z + w = 0$$

$$2x + 3y + z + w = 1$$

$$5x + 7y + 3z + 3w = 2$$

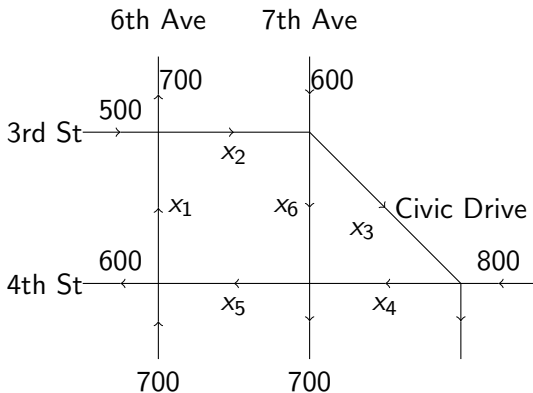
Tan, Chapter 2.3, Exercise 39 I

Mr. and Mrs. Garcia have a total of \$100,000 to be invested in stocks, bonds, and a money market account. The stocks have a rate of return (RoR) of 12%/year, the bonds pay 8%/year and the money market pays 4%/year. The Garcias have stipulated that the amount invested in stocks should be equal to the sum of the amount invested in bonds and 3 times the amount invested in the money market account. How should the Garcias allocate their resources if they require an annual income of \$10,000 from their investments? Give two specific options.

Tan, Chapter 2.3, Exercise 40 I

The accompanying figure shows the flow of traffic near a city's Civic Center during the rush hours on a typical weekday. Each road can handle a maximum of 1000 cars/hour without causing congestion. The flow of traffic is controlled by traffic lights at each of the five intersections.

Tan, Chapter 2.3, Exercise 40 II



- Set up a system of linear equations describing the traffic flow.
- Solve the system devised in part (a), and suggest two possible traffic-flow patterns that will ensure no traffic congestion.

Tan, Chapter 2.3, Exercise 40 III

- (c.) Suppose 7th Avenue, between 3rd and 4th street is soon to be closed for road repairs. Find one possible traffic-flow pattern that will result in a smooth flow of traffic.