MA162: Finite mathematics

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SCHEDULE:

- Fifth Web Assign assignment (Chapter 2.3) due on Friday, Septemeber 20 by 6:00 pm.
- Sixth Web Assign assignment (Chapter 2.4) due on Tuesday, September 24 by 6:00 pm.
- Exam 1 on Monday, September 30 from 5:00 pm to 7:00 pm.

Today we cover Chapter 2.4: Matrices. Over the last few lessons, matrices were used to save us from having to write down a bunch of variables. Here we'll see that matrices are much more than just a time-saving notational device.

Matrices

- A matrix is a rectangular array of numbers.
- The *size*, or *dimension*, of a matrix is the number of rows and number of columns.
- A 4×3 matrix:

$$\begin{bmatrix} 1 & -2 & 2 \\ 3 & 0 & 1 \\ 0 & 7 & 2 \\ 4 & 1 & 6 \end{bmatrix}$$

  First number refers to number of rows, second refers to number of columns.

- A *square matrix* has the same number of rows and columns:

$$\begin{bmatrix} 2 & 1 & 7 \\ 9 & 6 & 3 \\ 4 & 0 & 2 \end{bmatrix}$$

  A 3×3 square matrix.

When are two matrices equal?

- To be equal, two matrices must have the same *dimensions:*
  $\begin{bmatrix} 1 & -2 & 2 \\ 3 & 0 & 1 \\ 0 & 7 & 2 \\ 4 & 1 & 6 \end{bmatrix}$ and $\begin{bmatrix} 1 & -2 \\ 2 & 3 \\ 0 & 1 \\ 0 & 7 \\ 2 & 4 \\ 1 & 6 \end{bmatrix}$ can't be equal since they don't have same number of rows or columns.

- To be equal, the *entries* must be the same:
  $\begin{bmatrix} 1 & -2 & 2 \\ 3 & 0 & 1 \\ 0 & 7 & 2 \\ 4 & 1 & 6 \end{bmatrix}$ and $\begin{bmatrix} 3 & 0 & 1 \\ 4 & 1 & 6 \\ 1 & -2 & 2 \\ 0 & 7 & 2 \end{bmatrix}$ are not equal. Even though the dimensions are equal AND the matrices use the same numbers, the numbers appear in different positions.

- Don't confuse *equality* with *row equivalent*!

Matrix Equality

Find u and v so that the two matrices are equal:

$$\begin{bmatrix} 1 & 2u+3 & 2 \\ 3 & 0 & v \end{bmatrix} \quad \text{and} \quad \begin{bmatrix} 1 & 7-u & 2 \\ 2v & 0 & 1.5 \end{bmatrix}$$

Matrix addition

- Matrices can be added by adding corresponding entries:

$$\begin{bmatrix} 5 & 4 & 1 \\ 4 & 2 & -2 \end{bmatrix} + \begin{bmatrix} -1 & 2 & 0 \\ 3 & -1 & 3 \end{bmatrix} = \begin{bmatrix} 5-1 & 4+2 & 1+0 \\ 4+3 & 2-1 & -2+3 \end{bmatrix} = \begin{bmatrix} 4 & 6 & 1 \\ 7 & 1 & 1 \end{bmatrix}$$

- Can any two matrices be added? NO! Can only add matrices which have the same size.

$$\begin{bmatrix} 1 & 2 & 3 \end{bmatrix} + \begin{bmatrix} 1 & 2 \\ 2 & -1 \end{bmatrix} = \begin{bmatrix} 1+1 & 2+2 & 3+?? \\ ??+2 & ??-1 & ??+?? \end{bmatrix}$$

This sum doesn't make sense.

- Similarly, we subtract matrices entry by entry:

$$\begin{bmatrix} 5 & 4 \\ 9 & 6 \end{bmatrix} - \begin{bmatrix} 2 & 3 \\ 2 & -8 \end{bmatrix} = \begin{bmatrix} 5-2 & 4-3 \\ 9-2 & 6-(-8) \end{bmatrix} = \begin{bmatrix} 3 & 1 \\ 7 & 14 \end{bmatrix}$$

Scalar Multiplication

- Given a real number c and a matrix A, the *scalar product* of c and A, denoted cA, is obtained by multiplying c against each entry of A.

- If $c = 3$ and
$$A = \begin{bmatrix} 1 & 2 \\ 1 & 3 \\ 1 & -5 \end{bmatrix}$$
then
$$cA = 3\begin{bmatrix} 1 & 2 \\ 1 & 3 \\ 1 & -5 \end{bmatrix} = \begin{bmatrix} 3 \cdot 1 & 3 \cdot 2 \\ 3 \cdot 1 & 3 \cdot 3 \\ 3 \cdot 1 & 3 \cdot (-5) \end{bmatrix} = \begin{bmatrix} 3 & 6 \\ 3 & 9 \\ 3 & -15 \end{bmatrix}$$

- Scalar multiplication is defined for matrices of any size.

Familiar Laws, Unfamiliar Objects

- Matrix addition is commutative: provided A and B have the same dimensions

$$A + B = B + A$$

- Matrix addition is associative: provided A, B, and C have the same dimensions

$$A + (B + C) = (A + B) + C$$

- Scalar multiplication distributes over matrix addition: provided A and B have the same dimensions

$$c(A + B) = cA + cB$$

- A different kind of distributive law:

$$(c + d)A = cA + dA$$

Multiplication and Division?

- We've discussed how to add and subtract matrices
- We've see how to multiply a matrix by a real number
- Can we multiply two matrices together?
- Yes...but we'll wait until the next lesson to see how
- HINT: multiplying entry by entry is not the correct way to multiply matrices!

Tan, Chapter 2.4, Exercise 36 I

The following table gives the number of shares of certain corporations help by Leslie and Tom in their respective IRA accounts at the beginning of the year:

	IBM	GE	Ford	Wal-Mart
Leslie:	500	350	200	400
Tom:	400	450	300	200

Over the year, they added more shares to their accounts, as shown in the table:

	IBM	GE	Ford	Wal-Mart
Leslie:	50	50	0	100
Tom:	0	80	100	50

Tan, Chapter 2.4, Exercise 36 II

(a.) Write a matrix A giving the holdings of Leslie and Tom at the beginning of the year and a matrix B giving the shares they have added to their portfolios.

(b.) Find a matrix C giving their total holdings at the end of the year.

Tan, Chapter 2.4, Exercise 38 I

K & R Builders build three models of houses, M_1, M_2, and M_3, in three subdivisions I, II, and III located in three different areas of a city. The prices are given in the matrix A:

$$A = \begin{bmatrix} 340 & 360 & 380 \\ 410 & 430 & 440 \\ 620 & 660 & 700 \end{bmatrix}$$

(The rows represent subdivisions I, II, and III respectively, the columns represent M_1, M_2, and M_3, respectively.)
The new price schedule for next year, reflecting a uniform percentage increase in each house, is given by the matrix B:

$$B = \begin{bmatrix} 357 & 378 & 399 \\ 430.5 & 451.5 & 462 \\ 651 & 693 & 735 \end{bmatrix}$$

What was the percentage increase in the prices of the houses?

The numbers of three types of bank accounts on January 1 at the Central Bank and its branches are represented by matrix A:

$$A = \begin{bmatrix} 2820 & 1470 & 1120 \\ 1030 & 520 & 480 \\ 1170 & 540 & 460 \end{bmatrix}$$

(The rows represent *Main office*, *Westside branch*, and *Eastside branch* respectively, the columns represent *Checking accounts*, *Savings accounts*, and *Fixed deposit accounts*, respectively.)
The number and types of accounts opened during the first quarter are represented in matrix B, and the number and types of accounts closed during the same period are represented in matrix C. Thus

$$B = \begin{bmatrix} 260 & 120 & 110 \\ 140 & 60 & 50 \\ 120 & 70 & 50 \end{bmatrix} \quad \text{and} \quad C = \begin{bmatrix} 120 & 80 & 80 \\ 70 & 30 & 40 \\ 60 & 20 & 40 \end{bmatrix}$$

Tan, Chapter 2.4, Exercise 39 II

(a.) Find matrix D, which represents the number of each type of account at the end of the first quarter at each location.

(b.) Because a new manufacturing plant is opening in the immediate area, it is anticipated that there will be a 10% increase in the number of accounts at each location during the second quarter. Write a matrix $E = 1.1D$ to reflect this anticipated increase.

The Campus Bookstore's inventory of books is
Hardcover:
textbooks, 5280; fiction, 1680; nonfiction, 2320; reference, 1890
Paperback:
fiction, 2810; nonfiction, 1490; reference, 2070, textbooks, 1940

The College Bookstore's inventory is
Hardcover:
textbooks, 6340; fiction, 2220; nonfiction, 1790; reference, 1980
Paperback:
fiction, 3100; nonfiction, 1720; reference, 2710, textbooks, 2050

(a.) Represent Campus's inventory as a matrix A.

(b.) Represent College's inventory as a matrix B.

(c.) The two decide to merge. Write a matrix C that represents the total inventory of the newly amalgamated company.